The Chaotic Marriage of Physics and Finance

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SO, how did it all begin???

What did he see in her?

What did she see in him?

Results of this fusion

Two sons (one a physicist)

One chaotic PhD thesis

Graduate student in search of thesis

?????

!!!!!!

Why Not???

Once should have been enough....

There was that experience with

Linear Algebra

?§!@ç#&!!!

Well, … why not???

Appeal of Chaos Theory

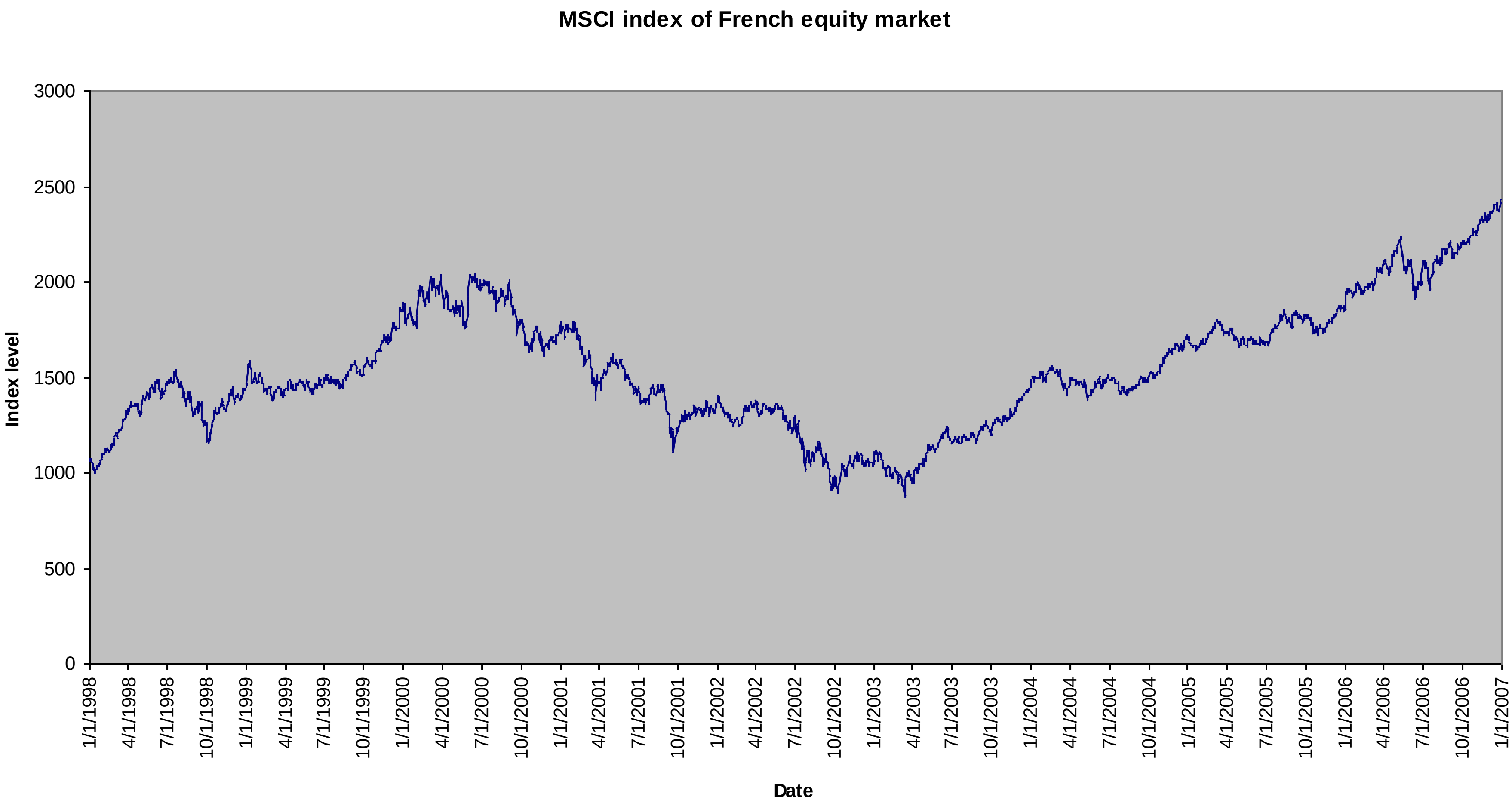
Complicated behavior, large and aperiodic

fluctuations, abrupt changes, in

Stock markets

Foreign exchange markets

Bond markets


MSCI index of French equity market
Index level
3000
2500
2000
1500
1000
500
0
1/1/1998
4/1/1998
7/1/1998
10/1/1998
1/1/1999
4/1/1999
7/1/1999
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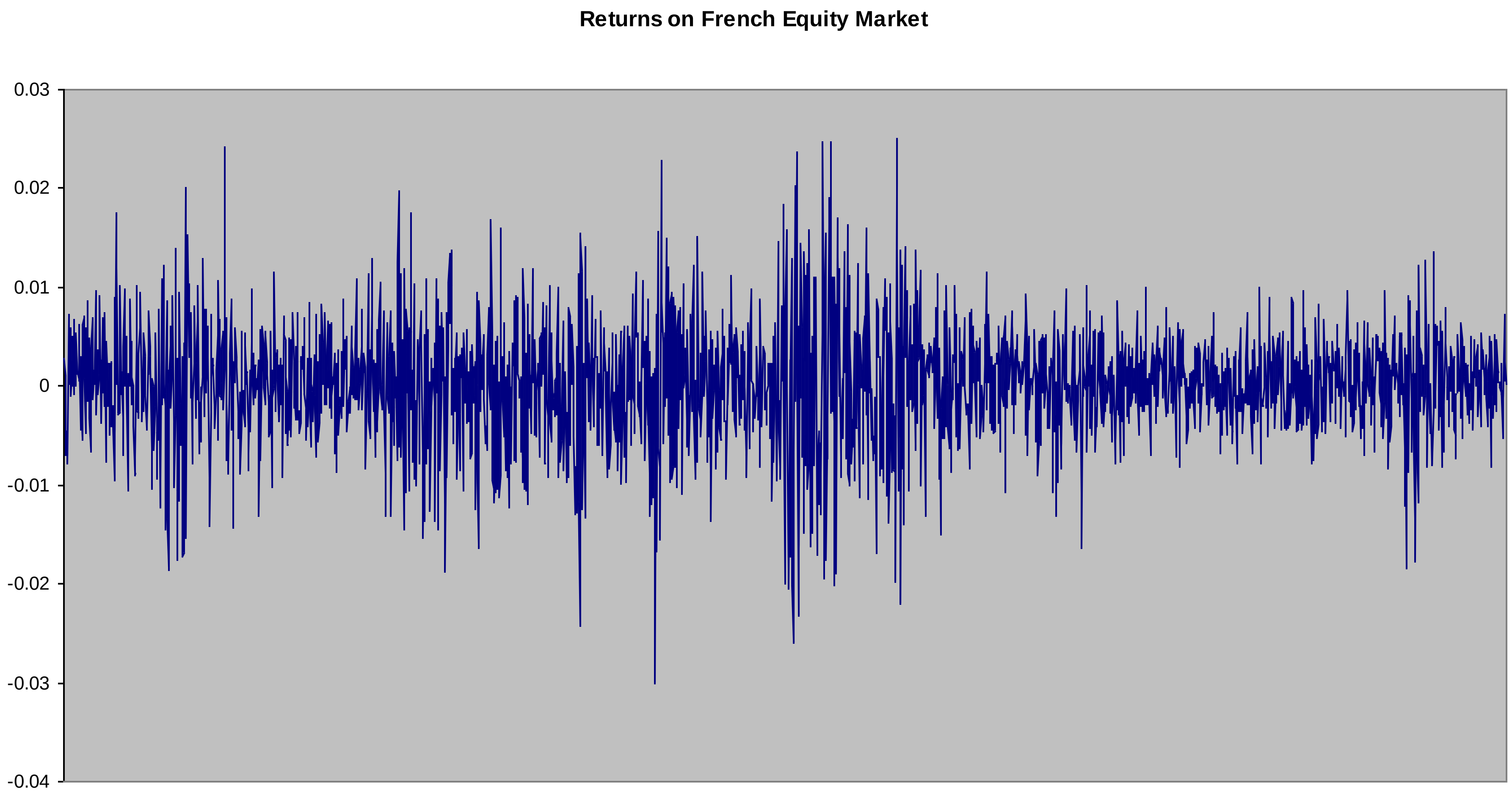
Date


Returns on French Equity Market
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04

Motivation

Financial market behavior still not well understood,

despite decades of intensive, sophisticated analysis.

Motivation

Traditional modeling:

treat fluctuations as linear perturbations near equilibrium
simple model of a geometric random walk with uncorrelated innovations.

This implies that stock returns, for example, are independent and identically distributed (iid) random variables.

Motivation

Development in 1980s of models to explain large fluctuations in volatility behavior. GARCH family of models.

Engle	1982
Bollerslev	1986

Motivation

Also in 1980s question arises: can chaos theory contribute to an understanding of financial market behavior?

Irregularities of economic and financial data are attributed by theory to random exogenous shocks, but even simple deterministic chaotic models can produce time paths that appear to be random.

SO INDEED, WHY NOT???

Bob does seem to have a better way....

Motivation

Some early applications of chaos theory by economists to economic and financial data claimed to find evidence of chaos using

- Correlation integral
- Lyapunov exponents

So you find chaos.... But then what?

A Better Methodology

A topological approach to the analysis of chaotic systems.

NOW ...

To learn about chaos theory!

Important Issue

A cautionary tale: make sure tools taken from one field to apply in another field are appropriate.

A test for chaos needs to be able to accommodate the peculiarities of economic and financial data series.

Peculiarities of Financial Data

1. Data set may be of limited size (max. $< 10^4$ obs.)
2. Presence of noise
3. Nonstationarity

The Close Returns Test

Based on the recurrence property, the tendency of the time series to nearly, although never exactly, repeat itself over time.

Searches for the unstable periodic orbits in a strange attractor.

Close Returns Test

Close Returns test: create a graph

$|x(t) - x(t+i)| < \varepsilon$ code black

$|x(t) - x(t+i)| > \varepsilon$ code white

Apply to data

Fixed ε

Arrange in easily readable horizontal form

Initial Work

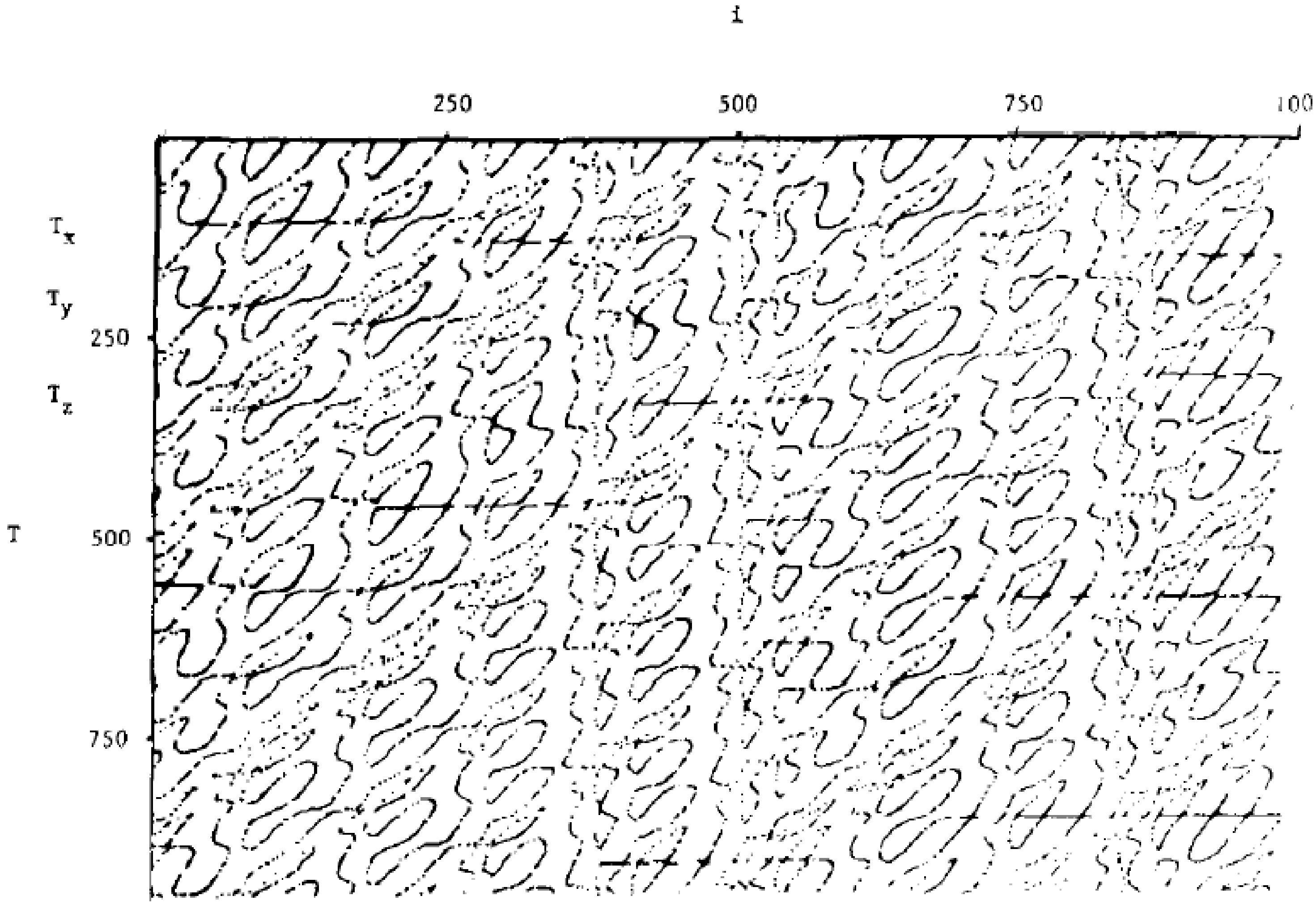
Generate a variety of times series data types:

(pseudo)random

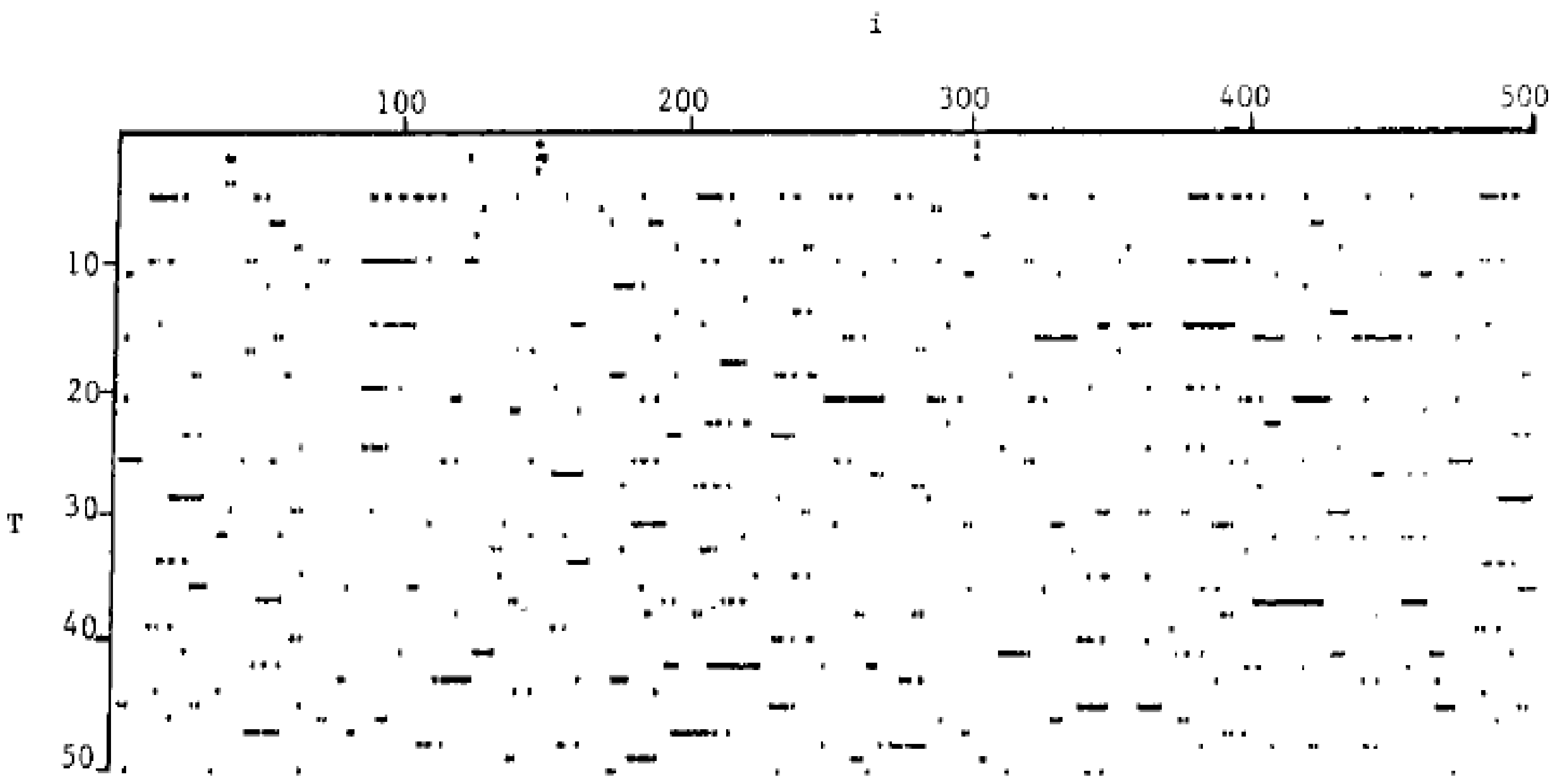
autoregressive

periodic

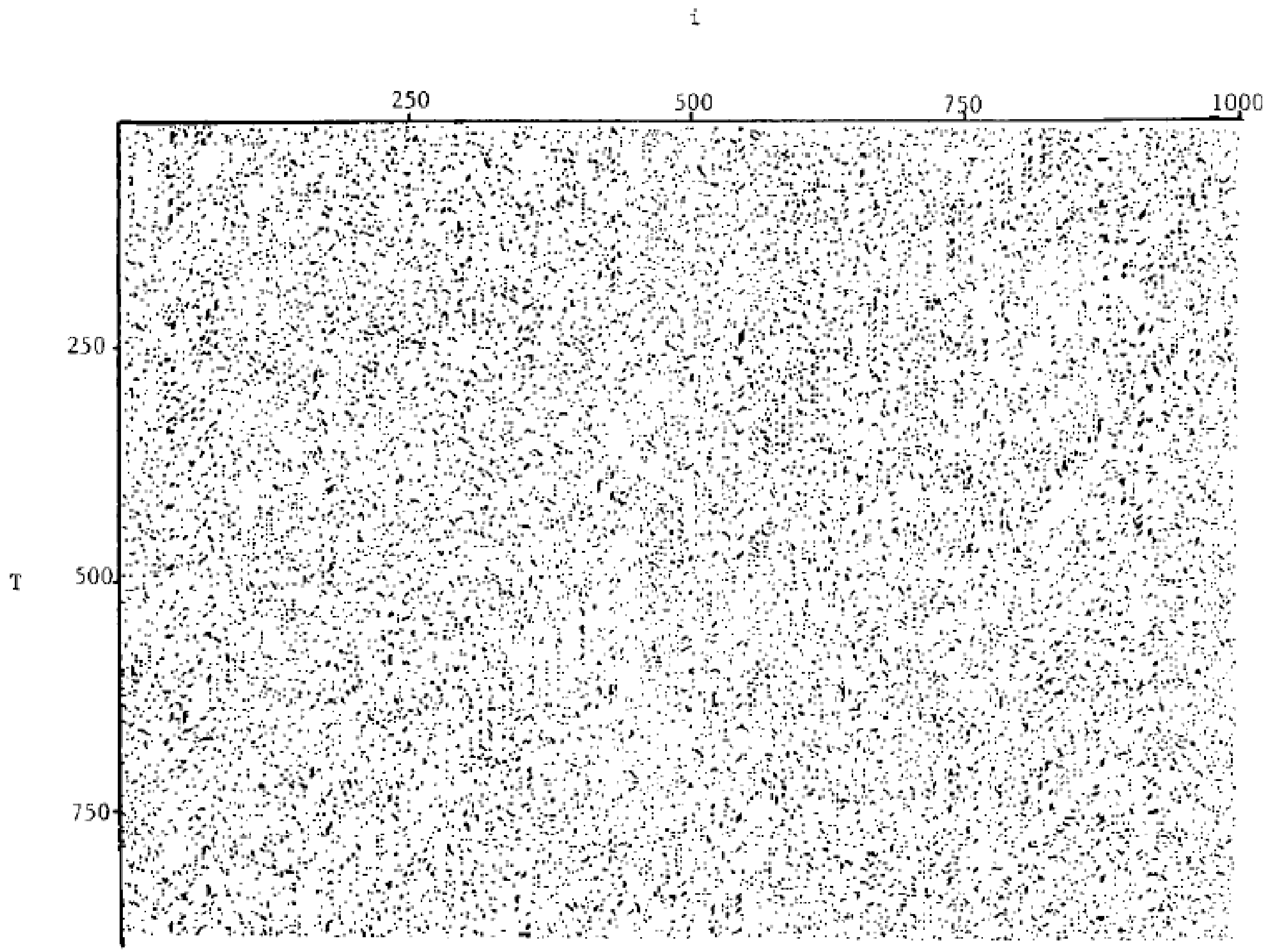
chaotic



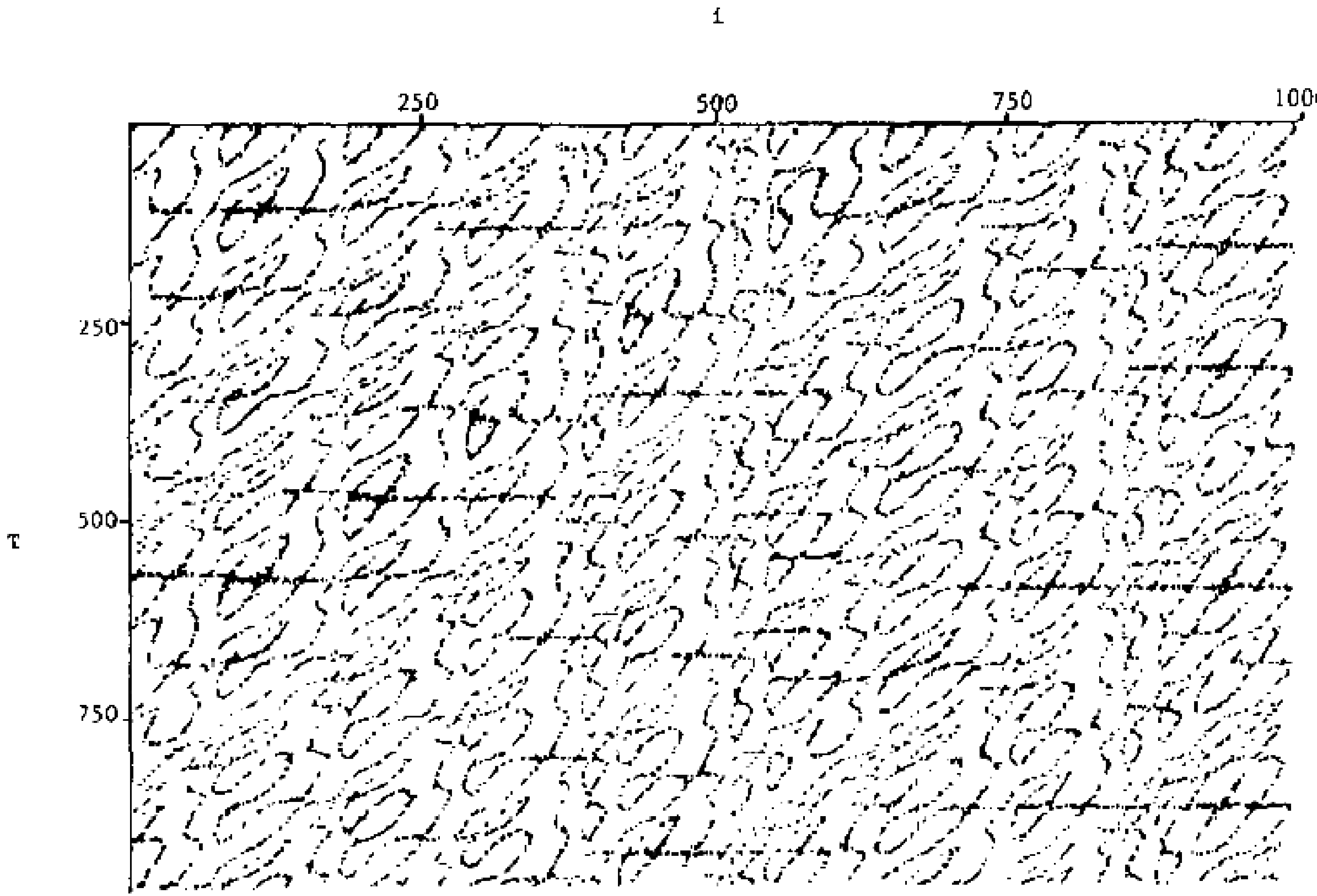
Close returns plot of chaotic time series, generated from Rössler model. First 1960 points of 5000-obervation set. Chaotic pattern present even with small number of observations.



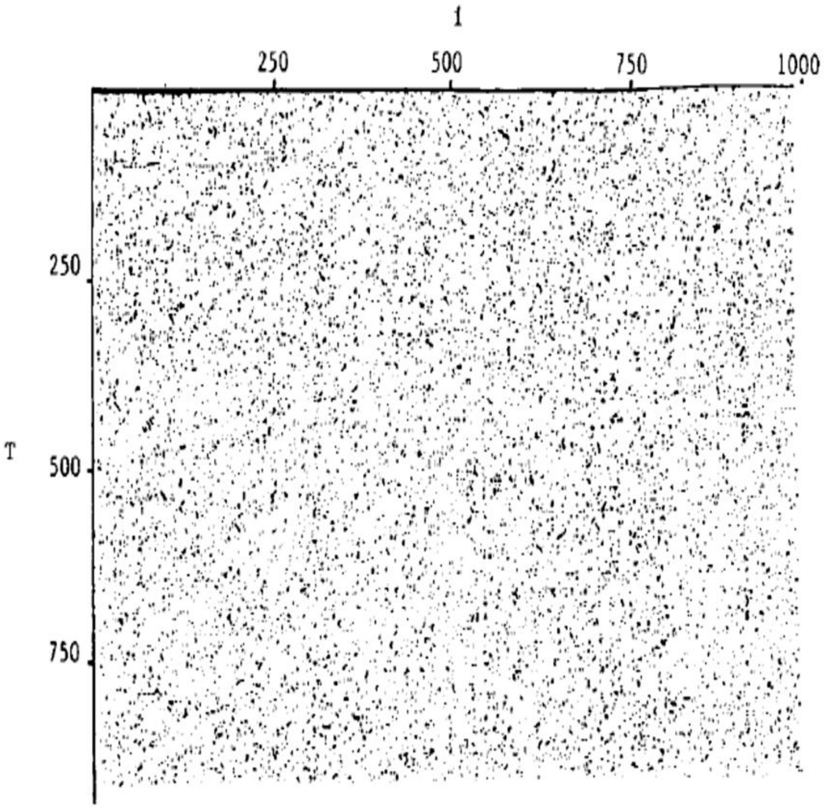
Close returns plot for logistic map: $X_{i+1} = \lambda X_i(1 - X_i)$. Parameter value: $\lambda = 3.75$. 500x50-observation plot.



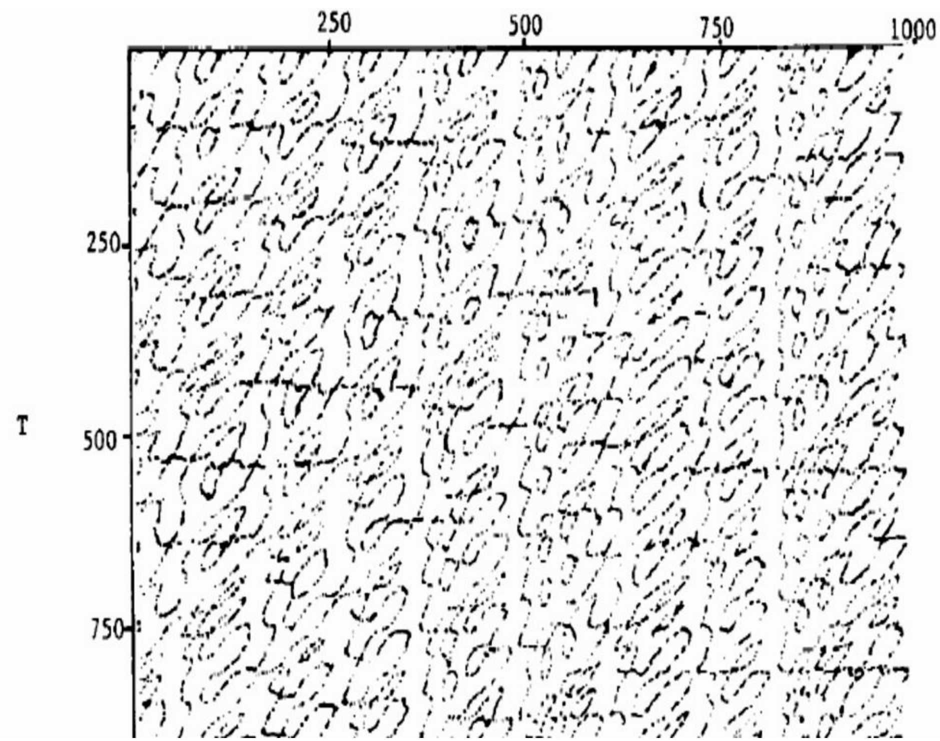
Close returns plot of pseudorandom time series.



Close returns plot of Rössler chaotic series with 15% addition of noise.
Chaotic pattern remains easily visible.



Rössler data
with 200% addition of noise



Rössler data
with 200% addition of noise;
Smoothed with 10-point MA

Chaotic signal is completely obscured by 200% addition of noise to Rössler series

Close Returns Test

1. Works on relatively small data sets.

2. Robust against noise.

3. Preserves time-ordering information.

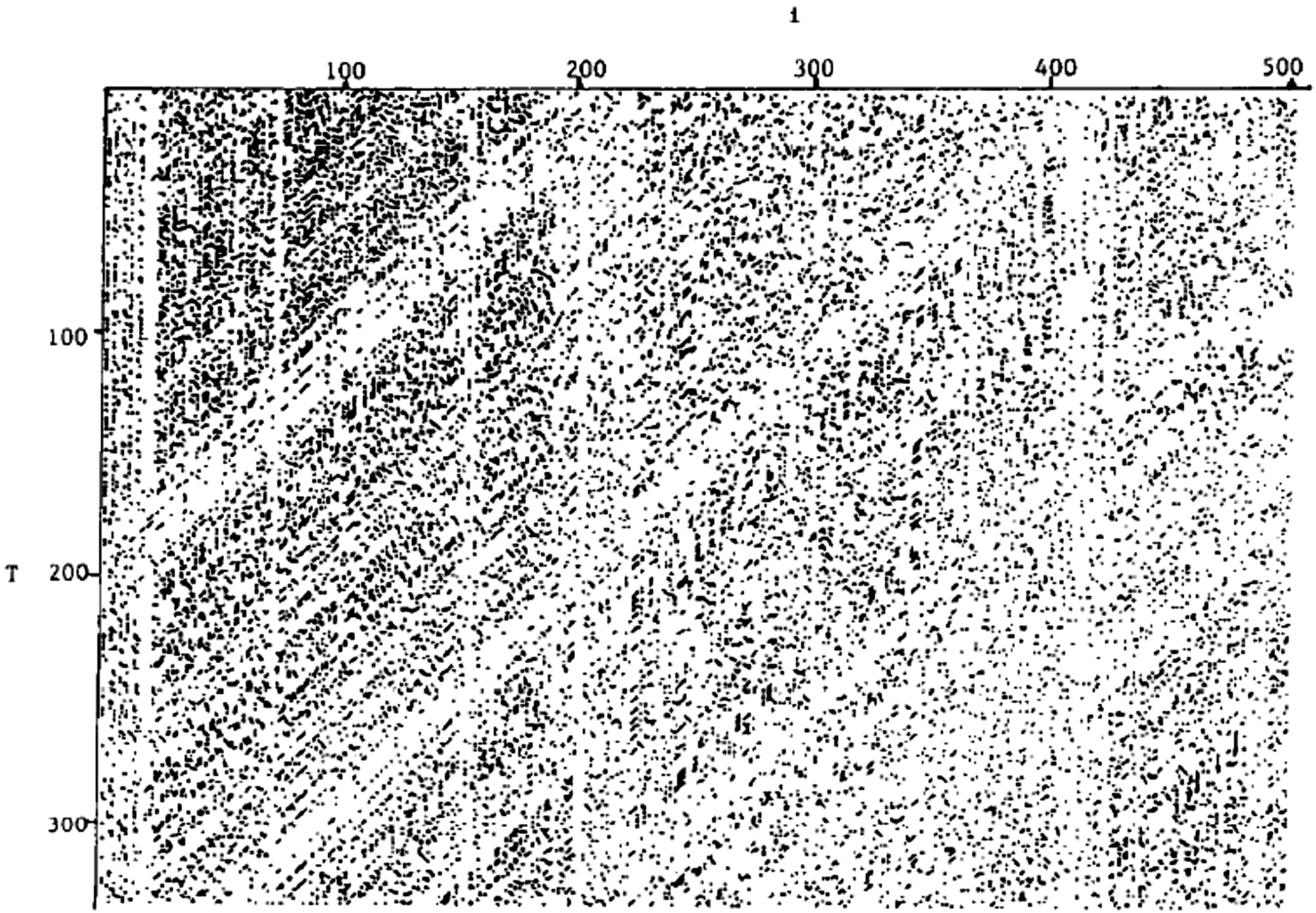
4. Can provide information about underlying system generating chaotic behavior, if evidence of chaos is detected.

Application of Close Returns Test to Stock Market Data

Standard and Poor’s 500 Index (S&P500)

Stock returns July 1962 – December 1989

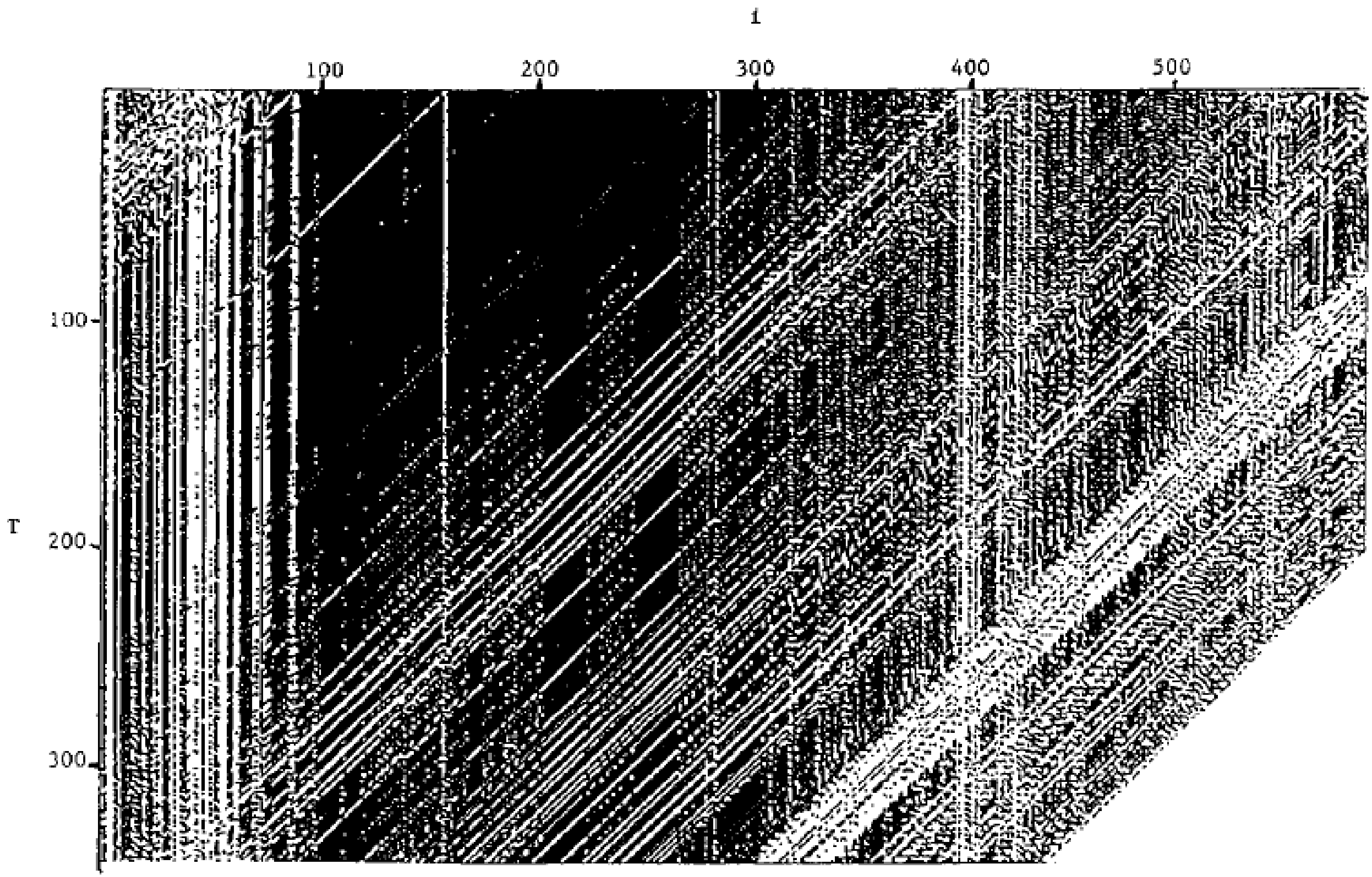
Weekly series, 1439 observations



Close returns plot of weekly stock returns.

Application of Close Returns Test to Treasury Bill Returns

780 monthly observations January 1926 – December 1990



Close returns plot of Treasury bill returns.

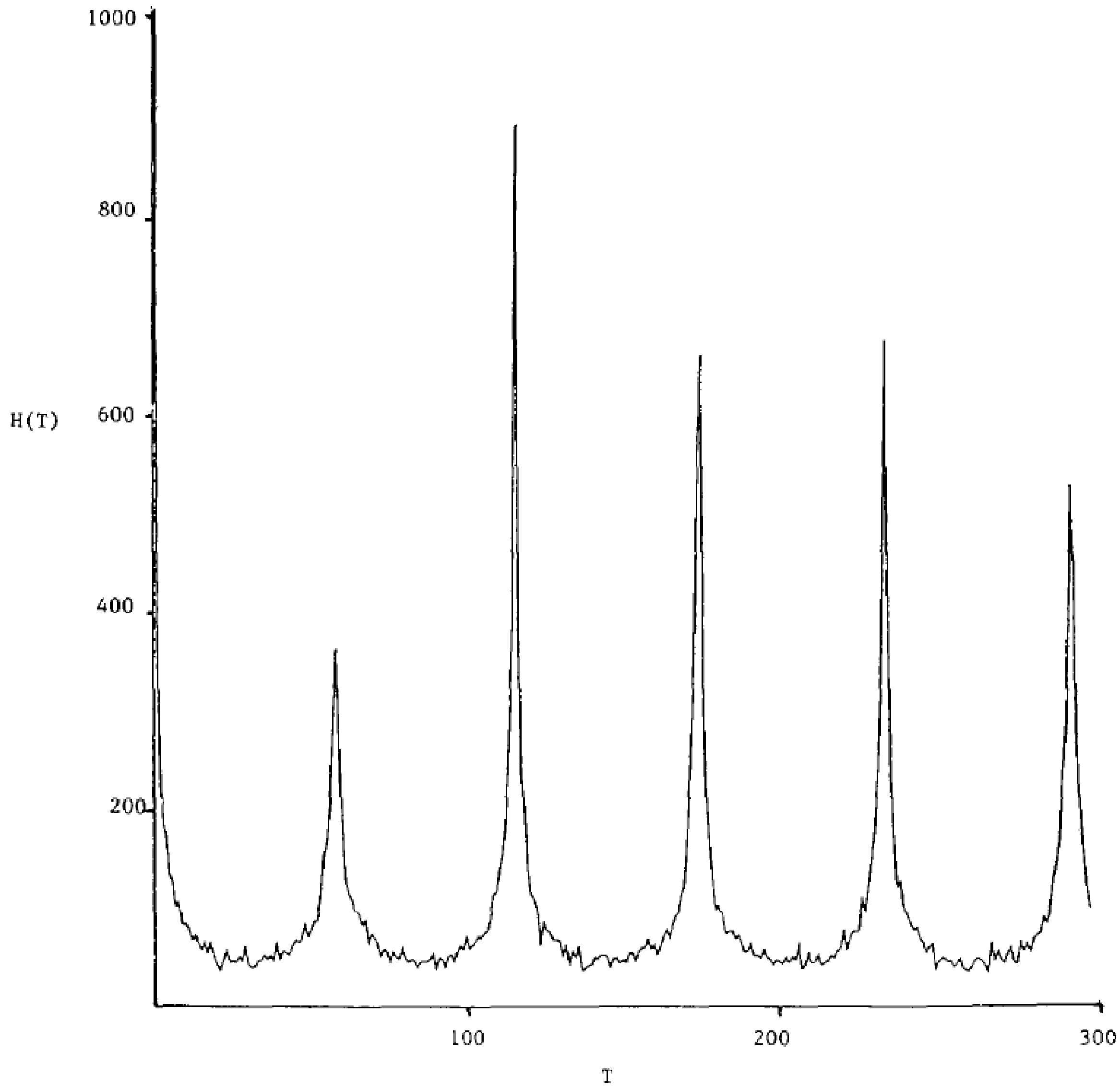
Disappointment

There's no chaos. Now try to get published:

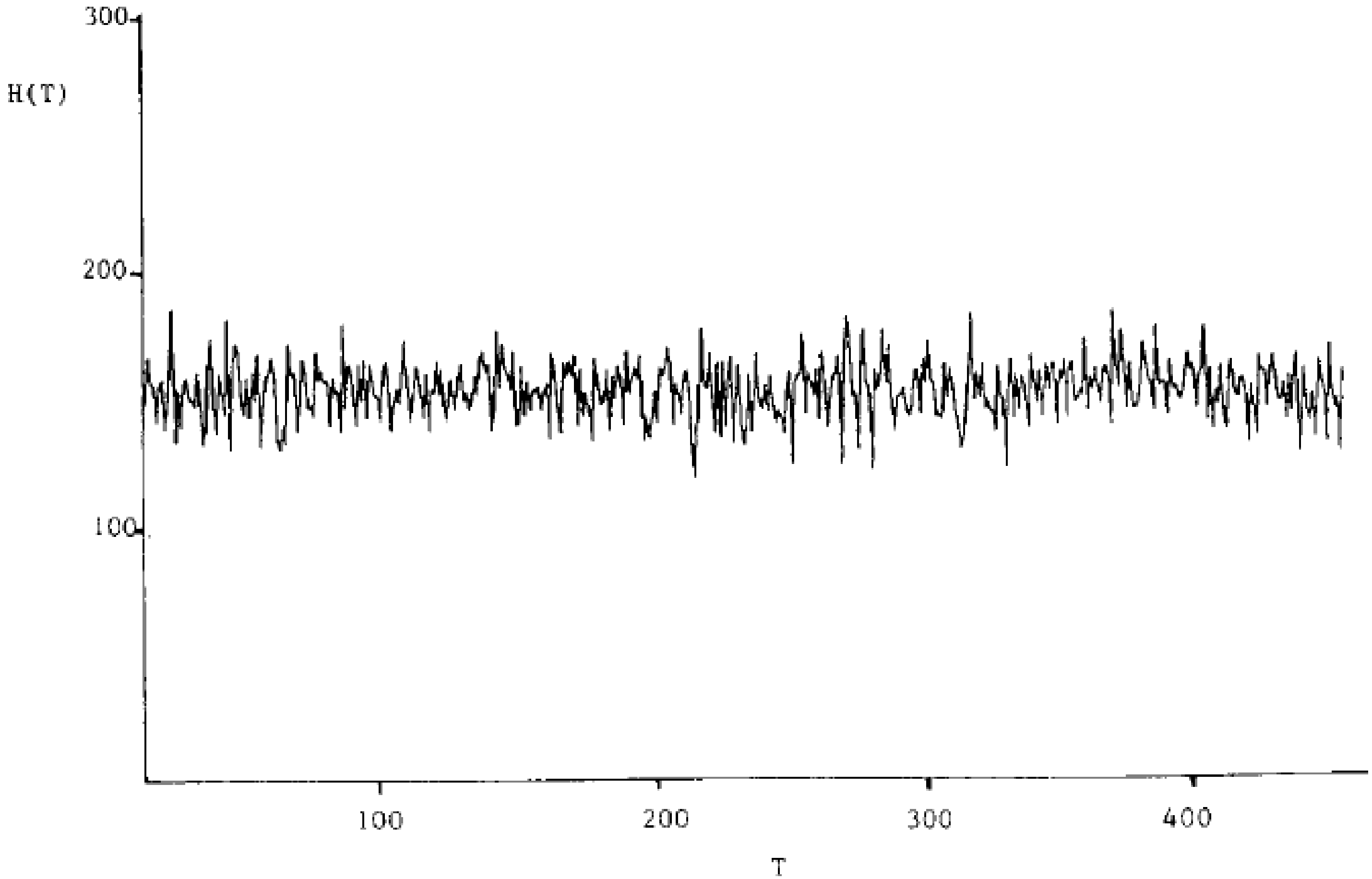
Later Development of Test

Quantitative close returns test:

$\chi 2$ test



Close returns histogram of Rössler chaotic series, using the full 5000-observation series over first 300 values of T.



Close returns histogram of pseudorandom data set.
Failure to reject H_0 of iid using χ_2 test.

Quantitative Close Returns Test: $\chi 2$ test

If the data are iid, then $H_i = \langle H \rangle$, a constant

The distribution will be binomial, with the probability of a hit , p,

$$p(\text{black}) = \frac{\text{total number of close returns}}{\text{total area of plot}}$$

and the average value is

$$\langle H \rangle = np$$

Quantitative Close Returns Test

$$\chi^2_c = \frac{\Sigma(H_i - \langle H \rangle)^2}{np(1-p)}$$

Null hypothesis: data are iid

Construct ratio of calculated-to-test statistic:

χ^2_t / χ^2_c if ratio > 1, reject null hypothesis of iid

Rejections of IID per Series

	100 obs.	500 obs.	1000 obs.
Uniform iid	46	52	58
Normal iid	46	28	24
Henon map	1000	1000	1000
Logistic map $\lambda = 3.65$	1000	1000	1000
Logistic map $\lambda = 3.65$	1000	1000	1000
AR(1), $\rho = 1.0$	963	1000	1000
AR(1), $\rho = 0.8$	514	875	958

Application of Close Returns Test to Stock Index Data

CRSP value-weighted return series (weekly)

July 2, 1962 – Dec. 31, 1989

$\chi2_2$ Ratio Test Results

Jan. 1976 – Dec. 1995	1.49
First third of observation set	1.0557
Second two thirds	1.0801

Conclusions

No positive evidence of chaos in exchange-rate data.

However, some nonlinear dependence persists, which is not adequately accounted for by standard GARCH and EGARCH models.

Why don't we find chaos in financial market data??

Theoretical models of financial markets can produce chaotic behavior:

Chiarelli	1990
Puu	1991
De Grauwe et Dewachter	1995
Chian et al.	2006

There are simply too many exogenous factors.

Is There Life (Marriage)After Chaos?

ECONOPHYSICS!!!!!

Motivation

Research looking for evidence of chaos in Empirical economic and financial data:

LeBaron 1988
Scheinkman and LeBaron 1989
Ramsey 1990
Brock et al 1991

Motivation

Many challenges to this type of model in 1980s:

Lo and MacKinlay	1988
Conrad and Kaul	1989
Fama and French	1988
Poterba and Summers	1998

Survey in Fama 1991

Motivation

Initial studies in economics and finance literature often claimed to find chaotic behavior or at least behavior consistent with chaos, using dimension calculations and Lyapunov exponents.

Later work tended to become more cautious, but even some recent papers (2004) have been published claiming compatibility with chaos, using these methods.

Short-term Forecasting

Use selected subsets of data from close returns plot

that closely match the sequence immediately preceding

the forecast period.

Short-term Forecasting

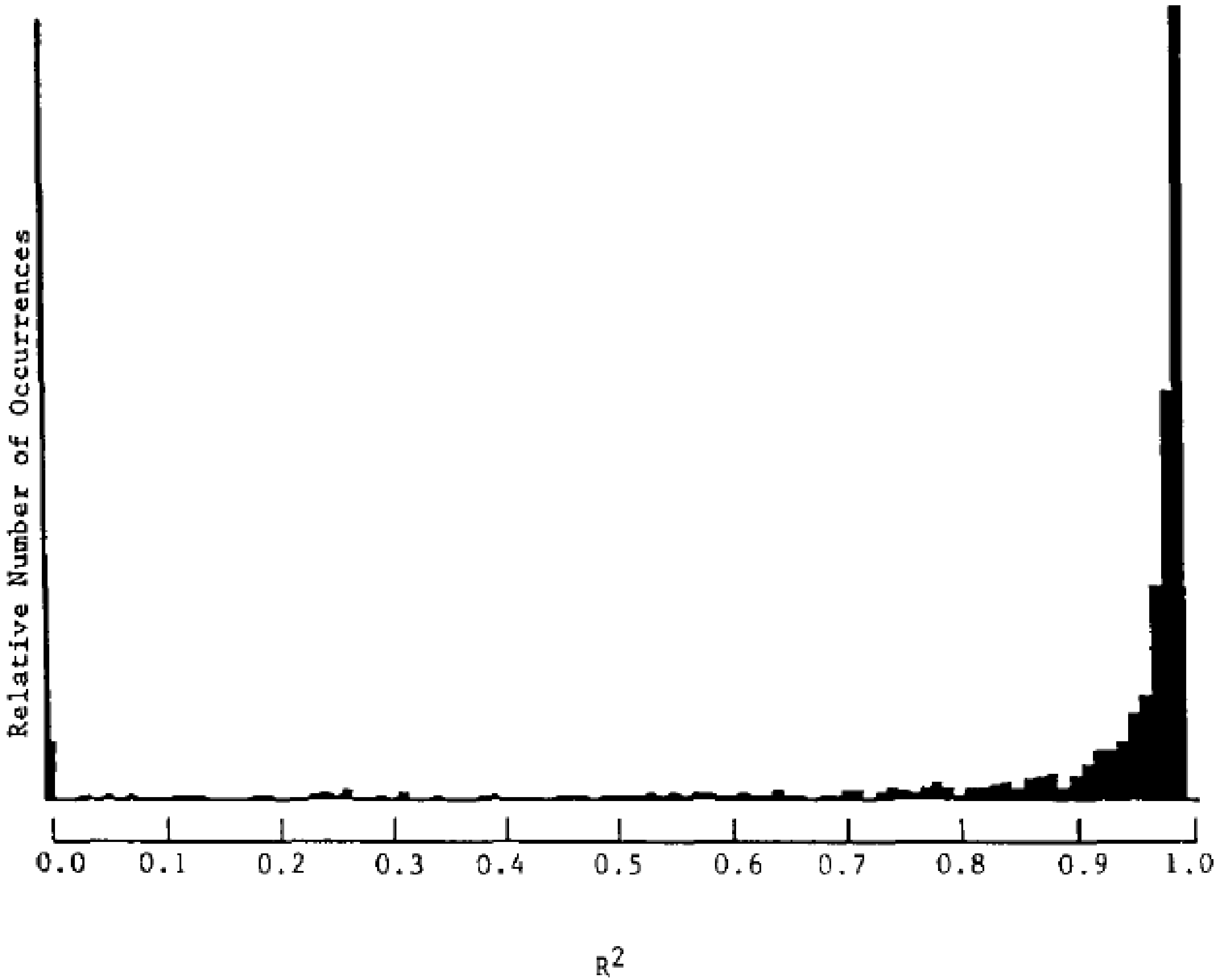
Procedure (illustrated with Roessler model data):

1. Select initial segment of x_i values (here 20 values)

2. Search across data for closely matching sequences

3. Extract the next 10 x_i values following each of the sequences identified in step 2.

4. Calculate weighted average values for each of the 10 x_i to serve as the short-term forecast.



Histogram of R^2 values for 2000 short-term predictions for chaotic time series.

Application of Close Returns Test to Exchange-Rate Data

British pound	Jan. 7, 1976 – Dec. 1, 1995
German Mark	Jan. 7, 1976 – Dec. 1, 1995
Japanese yen	Jan. 7, 1976 – June 17, 1994

5197 daily observations for pound and mark,

4814 for yen

χ_2 Ratio Test Results (After linear filter)

First third of observation set	1.038
Second two thirds	1.0327

χ_2 Ratio Test Results (After GARCH filter)

First third of observation set	0.7913
Second two thirds	0.959

Box Plot χ_2 Ratio Test

Original version of test emphasizes the regularly spaced horizontal line segments which results from chaos.

Other structure may get "washed out" from this one-dimensional projection.

Second version applies the test directly to the two-dimensional plot.

Box Plot χ_2 Ratio Test

Box size	Part 1		Part2	
	ε = 1%	ε = 2%	ε = 1%	ε = 2%
20	1.05	1.30	1.14	1.18
25	0.94	1.00	1.08	1.25
30	1.10	1.33	1.36	1.43
35	1.25	1.29	1.26	1.43
1	1.49	1.80	1.46	1.59
1	1.29	1.59	1.35	1.67
50	1.03	1.22	1.33	1.55

Box Plot χ_2 Ratio Test

Box size	Part 1		Part2	
	ε = 1%	ε = 2%	ε = 1%	ε = 2%
20	0.94	1.04	1.27	1.42
25	0.97	1.12	1.29	1.68
30	0.99	1.12	1.54	1.91
35	0.99	1.37	1.48	2.08
40	0.93	1.18	1.72	2.26
45	1.58	1.84	1.73	2.65
50	1.03	1.60	1.77	2.58

Box Plot χ_2 Ratio Test

Interpretation:

Some type of dependence, linear or nonlinear, is present in the exchange rate series.

Apply linear, nonlinear filters and test standardized residuals.

Box Plot χ_2 Ratio Test on Residuals

Linear filter:

$$x_t = b_o + b_1 D_{M,t} + b_2 D_{T,t} + b_3 D_{W,t} + b_4 D_{TH,t} + \Sigma b_i x_{t-i} + e_t$$

χ_2 ratio results for histogram:

pound = 1.211   mark = 1.013 yen = 0.965

χ_2 ratio results for box plot:

yen ratios now also exceed 1.0

Conclusion: indications of nonlinear dependence

Box Plot χ_2 Ratio Test

Nonlinear filter (GARCH(1,1) model):

$$x_t = b_o + b_1 D_{M,t} + b_2 D_{T,t} + b_3 D_{W,t} + b_4 D_{TH,t} + \Sigma b_i x_{t-i} + e_t$$

where e_t is normally distributed, with zero mean and variance h_t, such that

$$h_t = \gamma_o + \gamma_1 h_{t-1} + \phi e_{t-12}$$

χ_2 ratio results for box plot: evidence of nonlinearity for all series

Box Plot χ_2 Ratio Test

Nonlinear filter (exponential GARCH(1,1) model):

χ_2 ratio results for box plot: evidence of nonlinearity for all series

GARCH and EGARCH models do not adequately

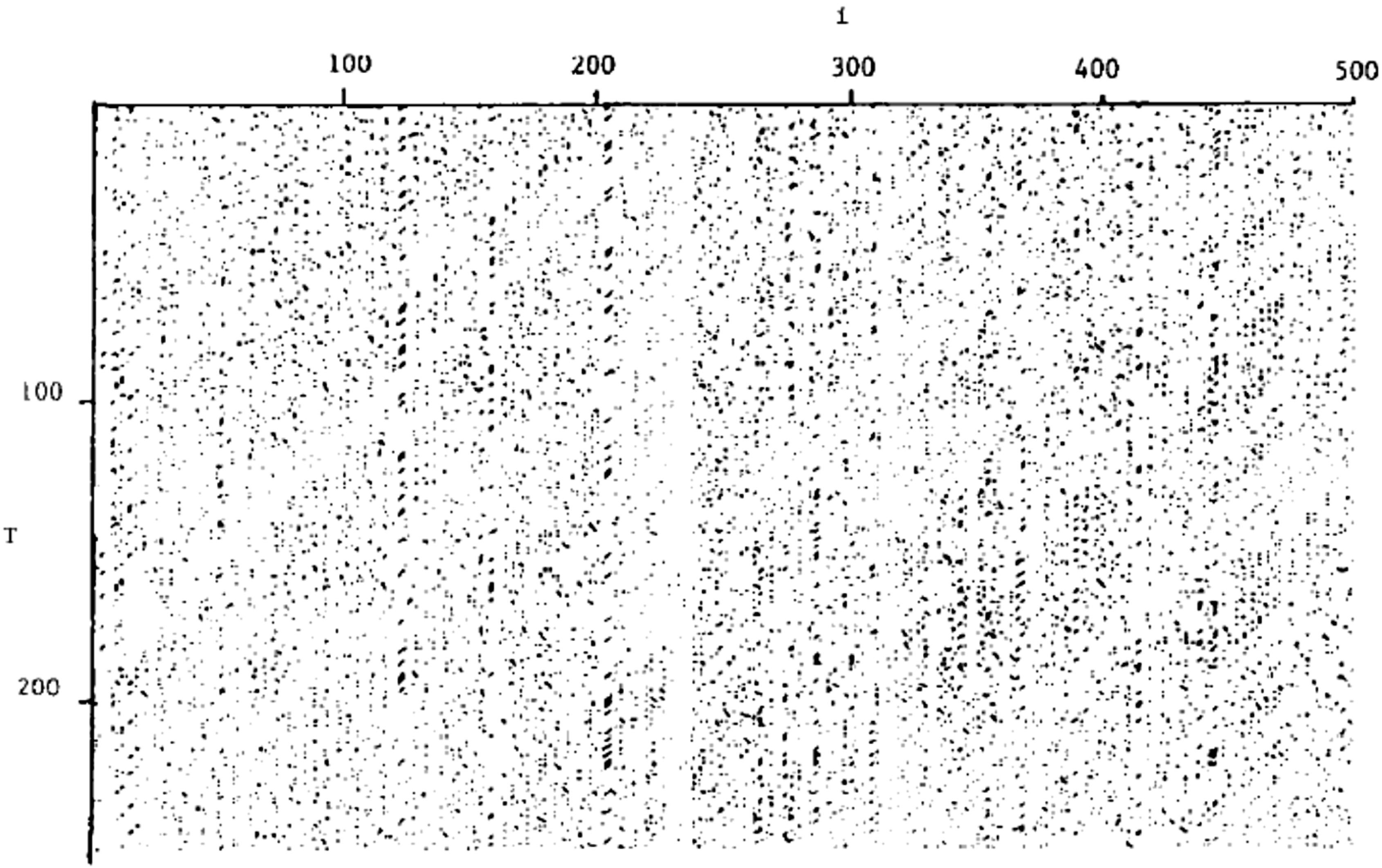
capture the nonlinearity in exchange rates

Application of Close Returns Test to Economic Data

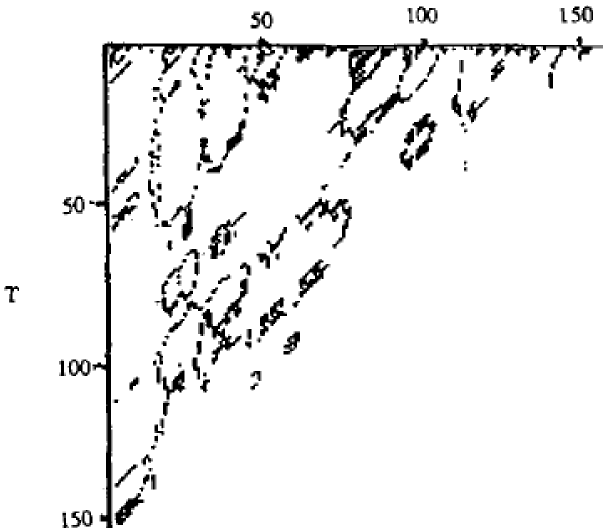
1. Absence of unstable periodic orbits rules out chaos.

1. Data are clearly not random, also not strictly periodic.

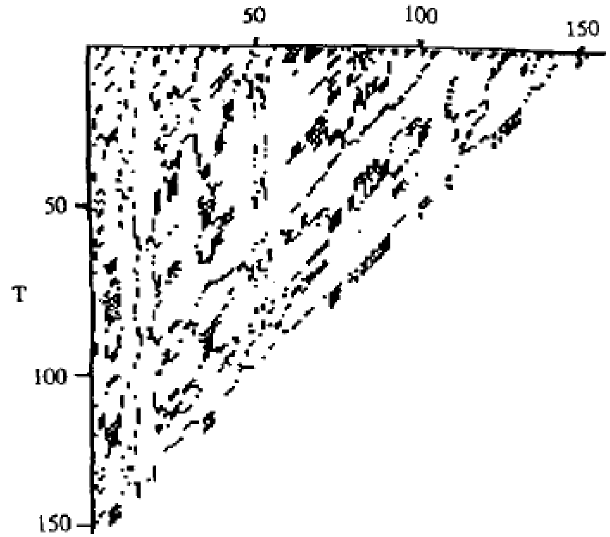
1. Some type of structure is present. Contour-like structure may be evidence of quasi-periodic behavior, of either a linear or a nonlinear origin. (Examination of Fourier spectrum of each series rejected hypothesis that the data are quasi-periodic.)



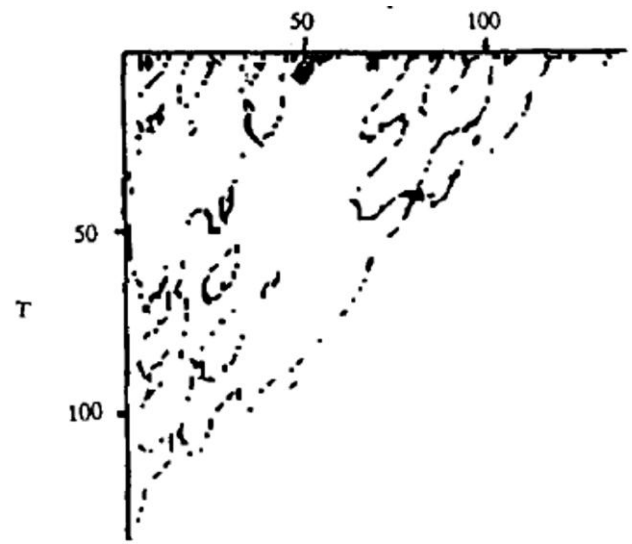
Close returns plot of German Mark – US dollar exchange rate



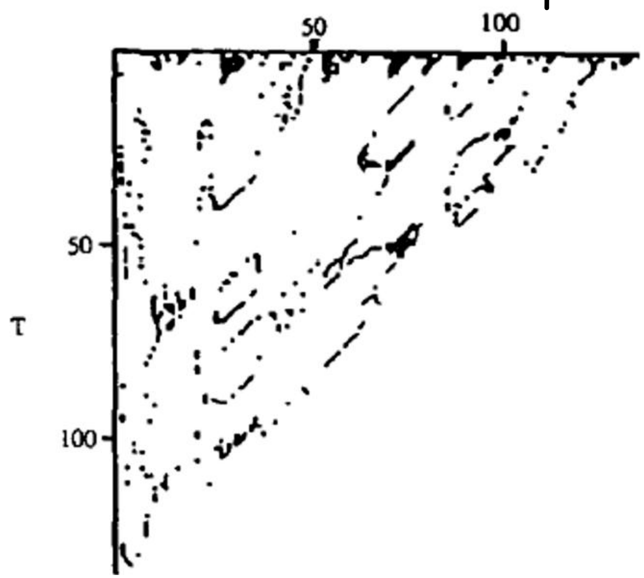
Gross national product



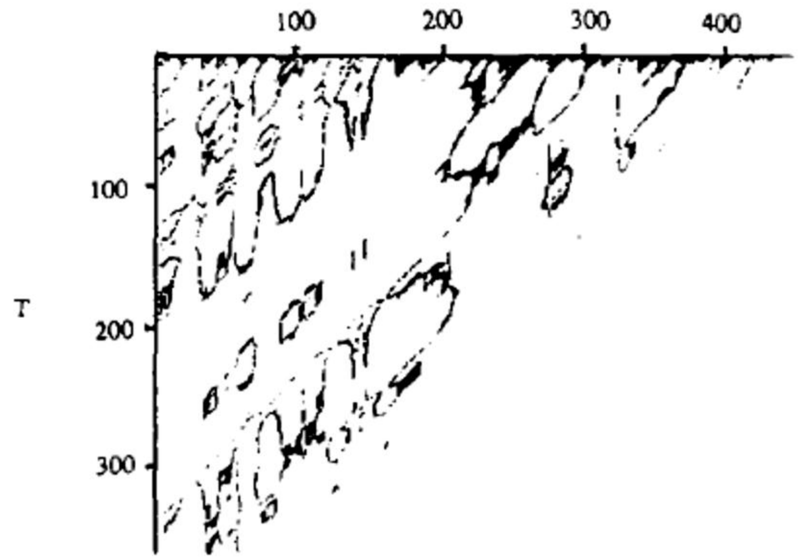
Gross domestic private investment



Employment



Unemployment rate



Industrial production index

Close returns plots of U.S. postwar macroeconomic series.

Objective

Demonstrate applications of several methods derived from the physics and mathematics literature to analysis of financial data.

Chaos theory

Singular value decomposition

Minimum spanning trees

Application of Close Returns Test to Economic Data

Business cycle data: quarterly or monthly, detrended,

133-432 observations.

Correlation dimension tests had produced some weak

evidence of low dimension in published work.

Other Mathematical Techniques of Current Interest

Singular Value Decomposition

Minimum Spanning Tree Analysis

Other Mathematical Techniques of Current Interest

Relevant to study of comovements of financial markets,

particularly equity markets, at international level.

- relevant to investors

- relevant to policymakers

Singular Value Decomposition

Explain the variability of returns in a system of equity

markets.

Germany, UK, Czech Republic, Hungary, Poland

1995-2005.

Singular Value Decomposition

X = N x p matrix of daily returns (approx. 2000) for 5 equity markets.

$X = U\Lambda V'$

Singular Value Decomposition

X = N x p matrix of daily returns (approx. 2000) for 5 equity markets.

$X = U\Lambda V'$

λ

2000 x 5

5 x 5

5 x 5

SVD COMPUTATION

Compute the p x p square matrix

$\mathbf{X'X} = (\mathbf{U\Lambda V'})'(\mathbf{U\Lambda V'}) = \mathbf{V\Lambda_2 V'}$

Diagonalizing $\mathbf{X'X}$ gives the spectrum of p eigenvalues $\lambda_{\alpha 2}$

and for each $\lambda_{\alpha 2}$ the eigenvector $v_{j,\alpha}$.

The dual eigenvectors $u_{i,\alpha}$ can be computed by diagonalizing

$\mathbf{X'X} = (\mathbf{U\Lambda V'})(\mathbf{U\Lambda V'})' = \mathbf{U\Lambda_2 U'}$.

Figure 2: Total Return Variability and Proportion due to Each Principal Component, 500-Day Rolling-Window

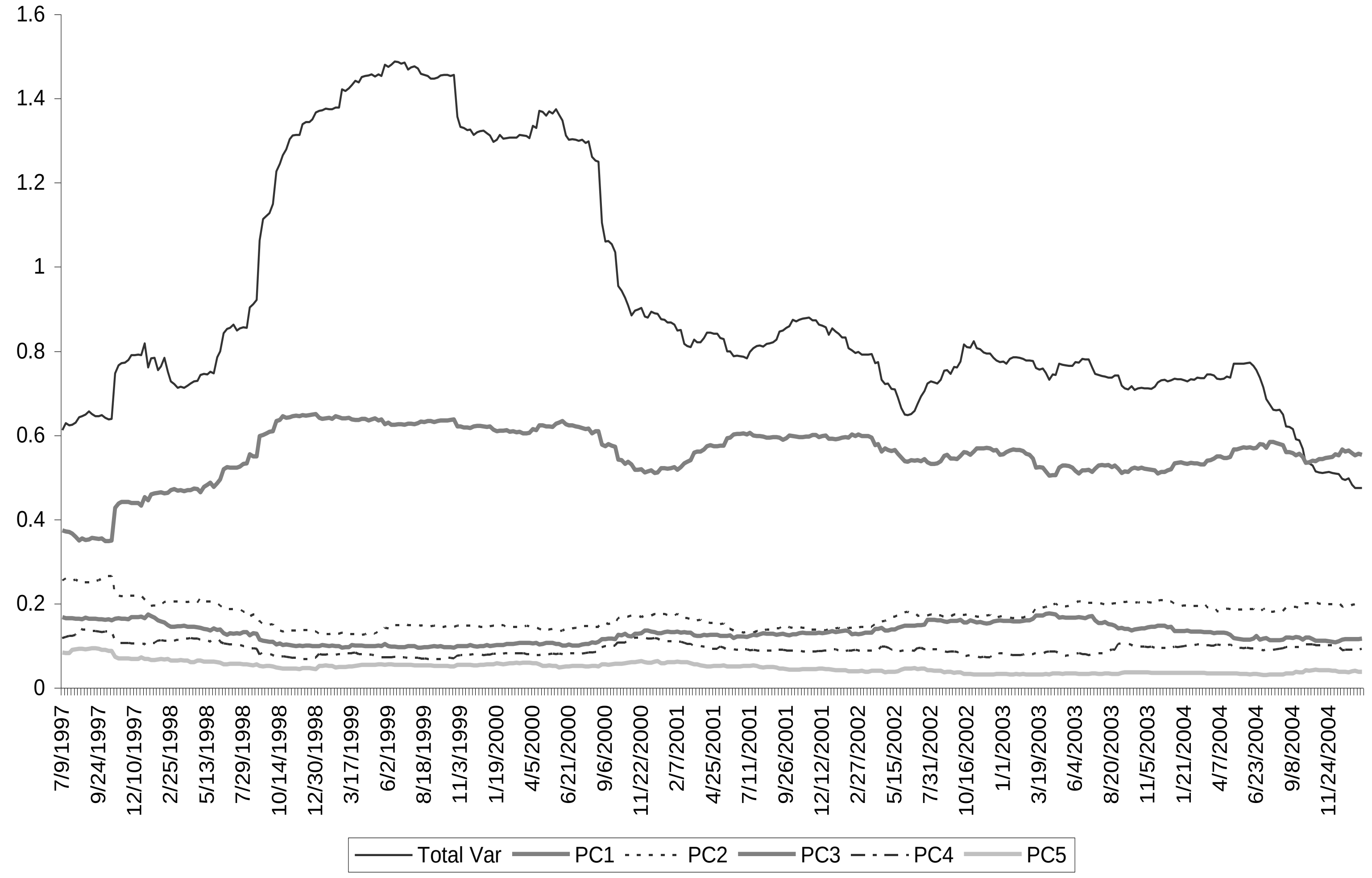


Figure 3a: Correlation of UK's Returns with the First Principal Component, 500-Day Rolling-Window

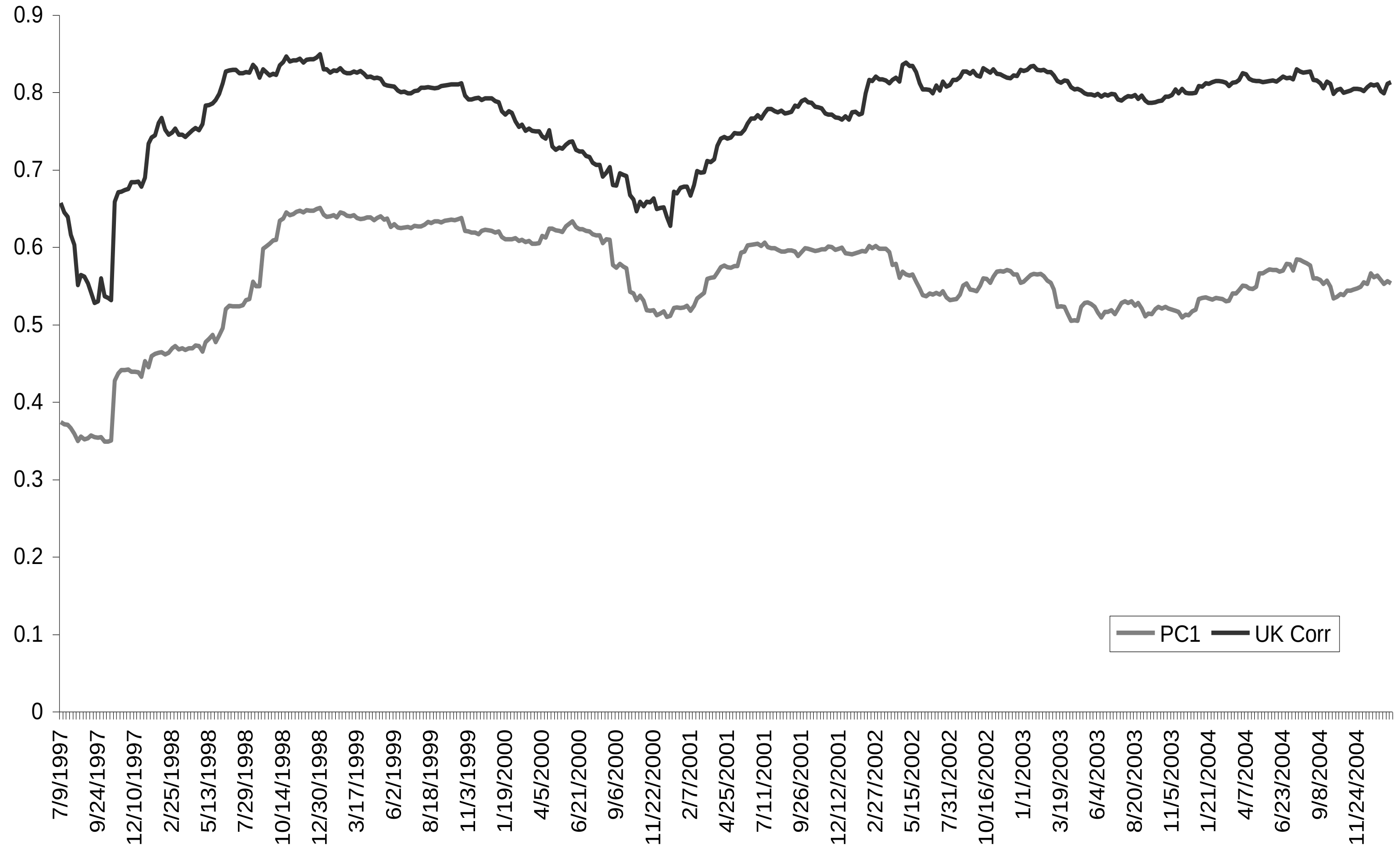


Figure 3b: Correlation of Germany's Returns with the First Principal Component, 500-Day Rolling-Window

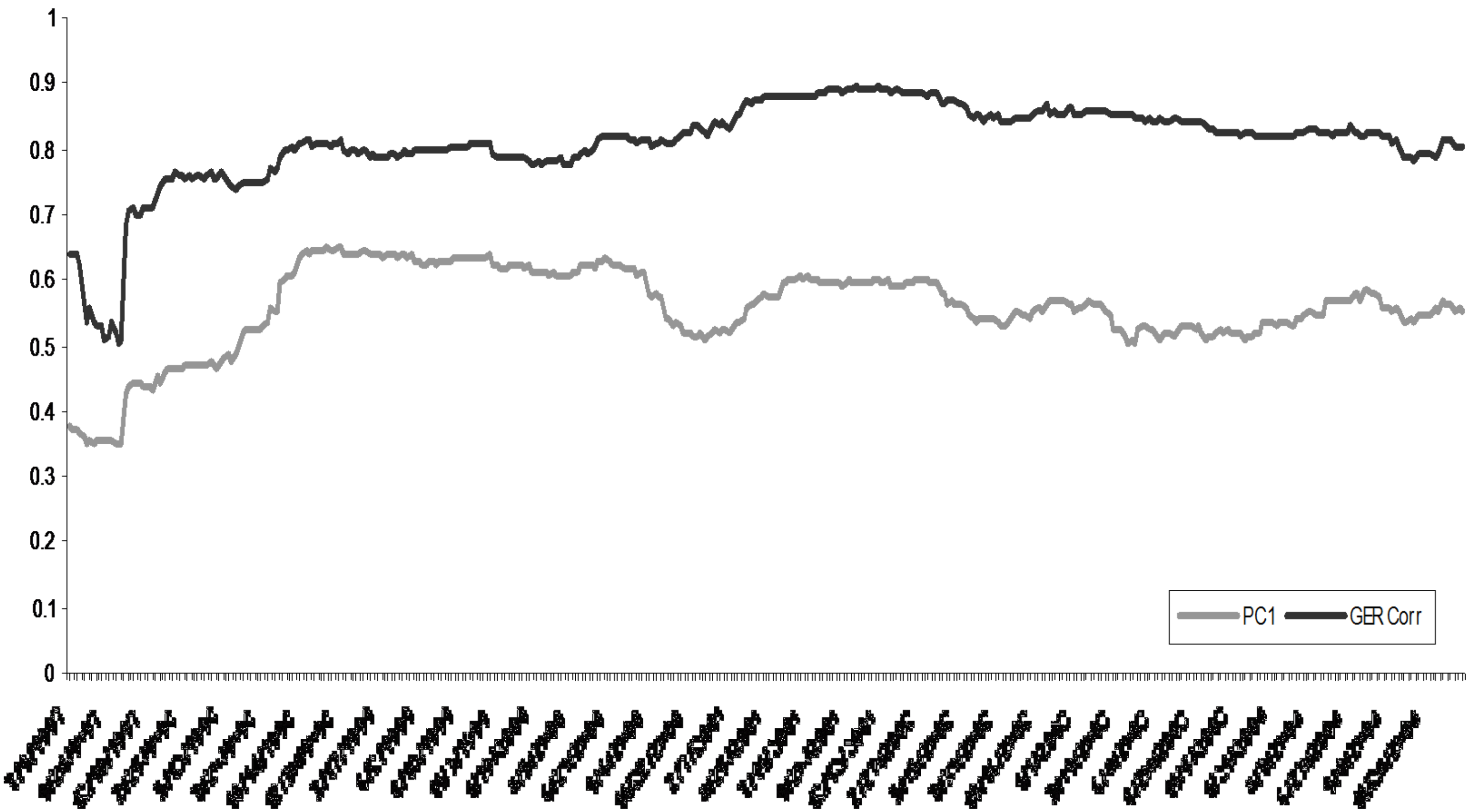


Figure 3c: Correlation of the Czech Republic's Returns with the First Principal Component, 500-Day Rolling-Window

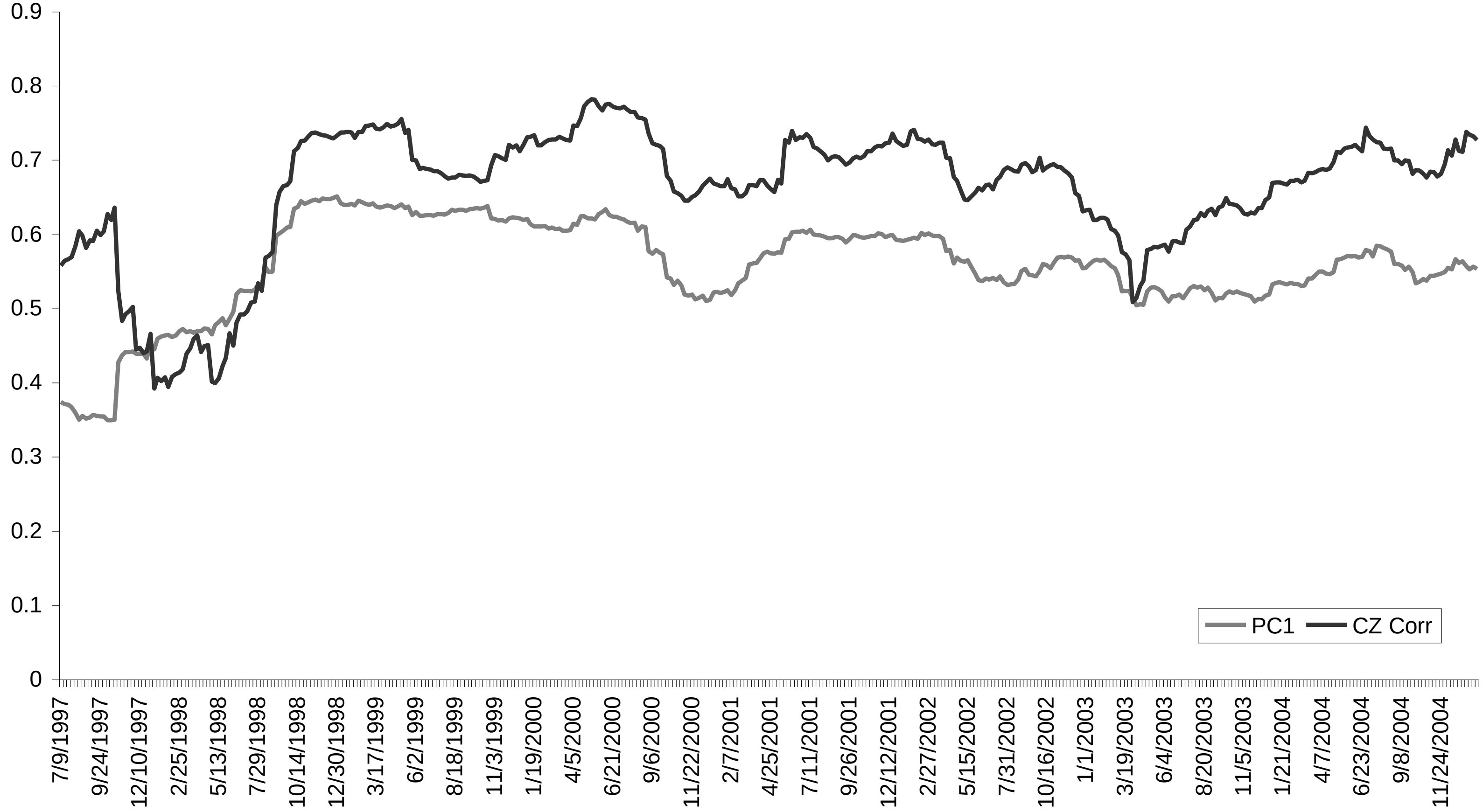


Figure 3d: Correlation of Hungary's Returns with the First Principal Component, 500-Day Rolling-Window

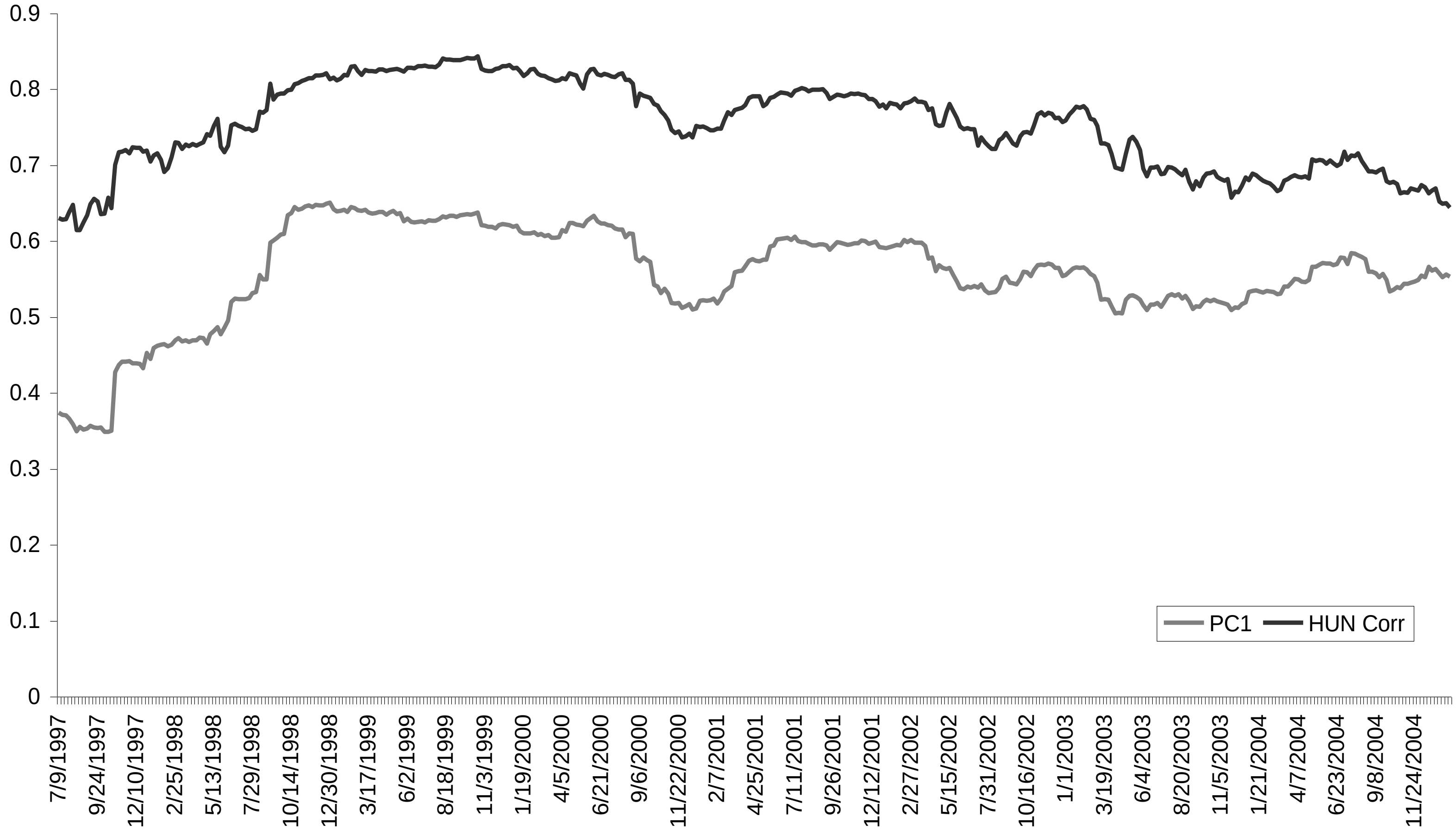
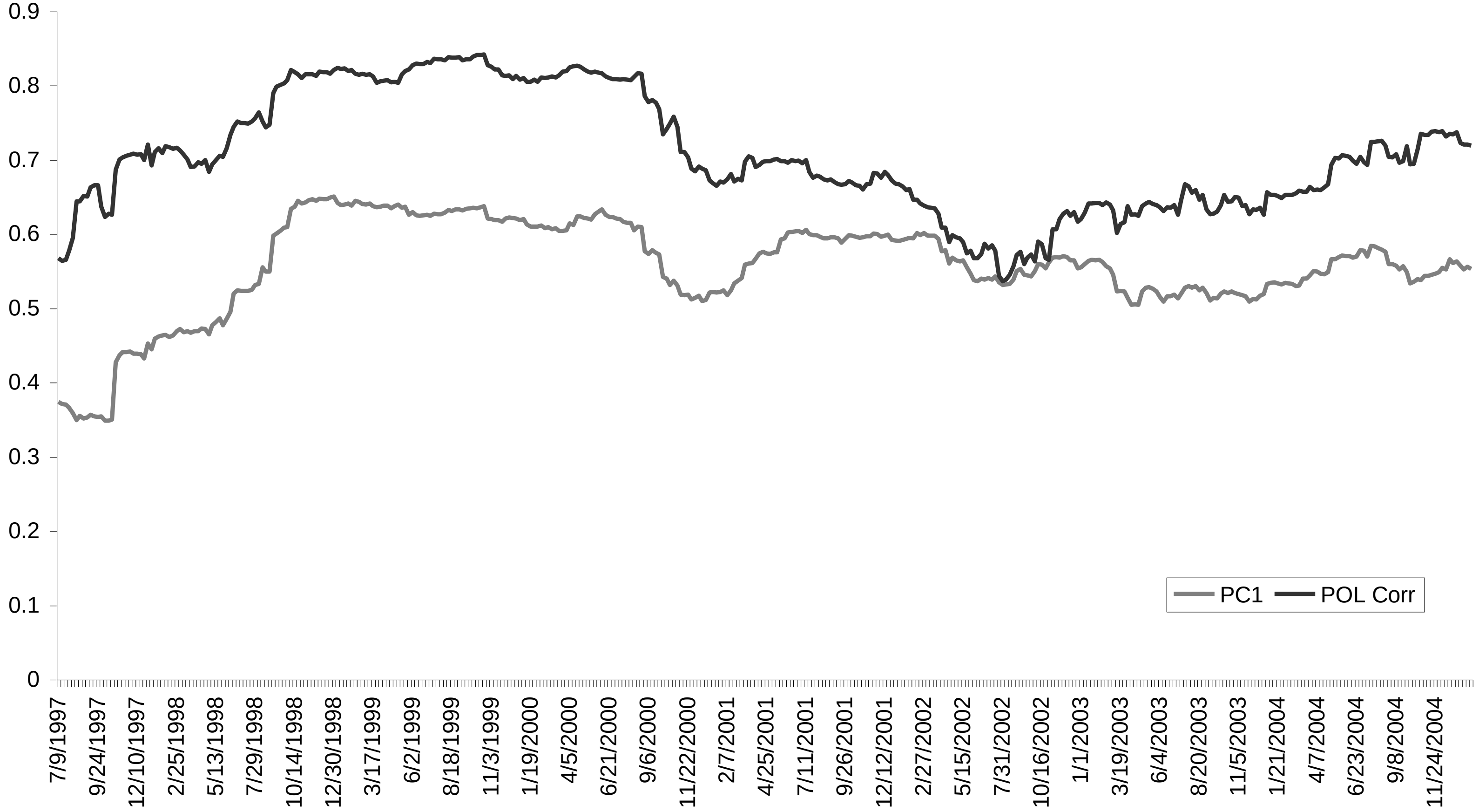
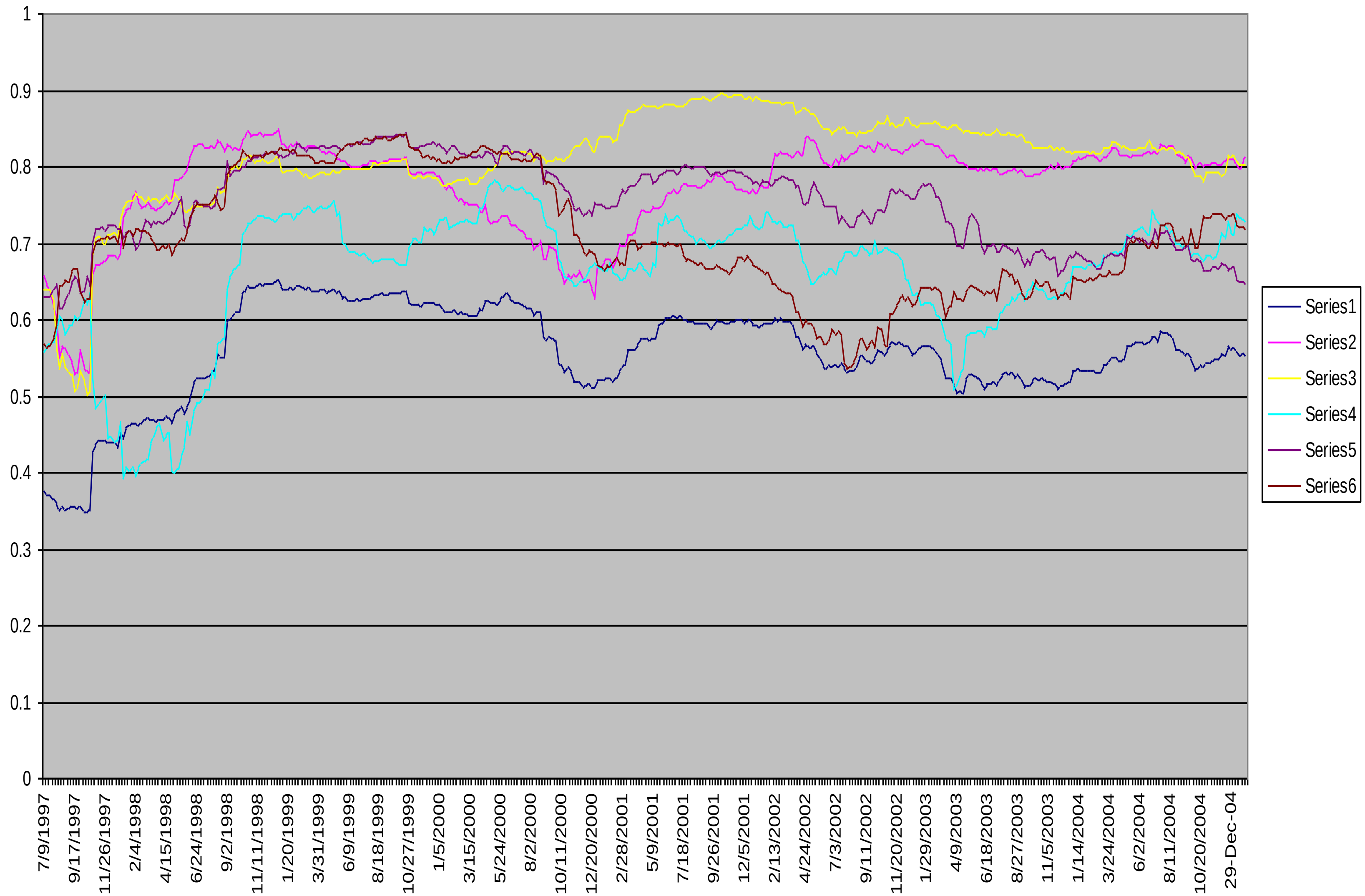


Figure 3e: Correlation of Poland's Returns with the First Principal Component, 500-Day Rolling-Window





1
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0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
7/9/1997
9/17/1997
11/26/1997
2/4/1998
4/15/1998
6/24/1998
9/2/1998
11/11/1998
1/20/1999
3/31/1999
6/9/1999
8/18/1999
10/27/1999
1/5/2000
3/15/2000
5/24/2000
8/2/2000
10/11/2000
12/20/2000
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5/9/2001
7/18/2001
9/26/2001
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2/13/2002
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7/3/2002
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1/29/2003
4/9/2003
6/18/2003
8/27/2003
11/5/2003
1/14/2004
3/24/2004
6/2/2004
8/11/2004
10/20/2004
29-Dec-04
Series1
Series2
Series3
Series4
Series5
Series6

Singular Value Decomposition

1. Large first eigenvalue indicates substantial degree of market linkages, of markets being driven by a common factor.

1. Little evidence of increase over recent years, so diversification benefits not likely reduced. Consistent with other studies showing equity markets lag behind.

1. Spread of correlations with first eigenvector reduced; three groupings of the 5 markets.

Minimum Spanning Trees

Drawn from graphing theory.

Way to study dynamical evolution of interdependence between

equity markets.

MSTs are connected graphs in which each node represents an

Equity market. Connected with the N – 1 most important links

Such that no loops are created.

Resume of Procedures used for Noise and Nonstationarity Problems

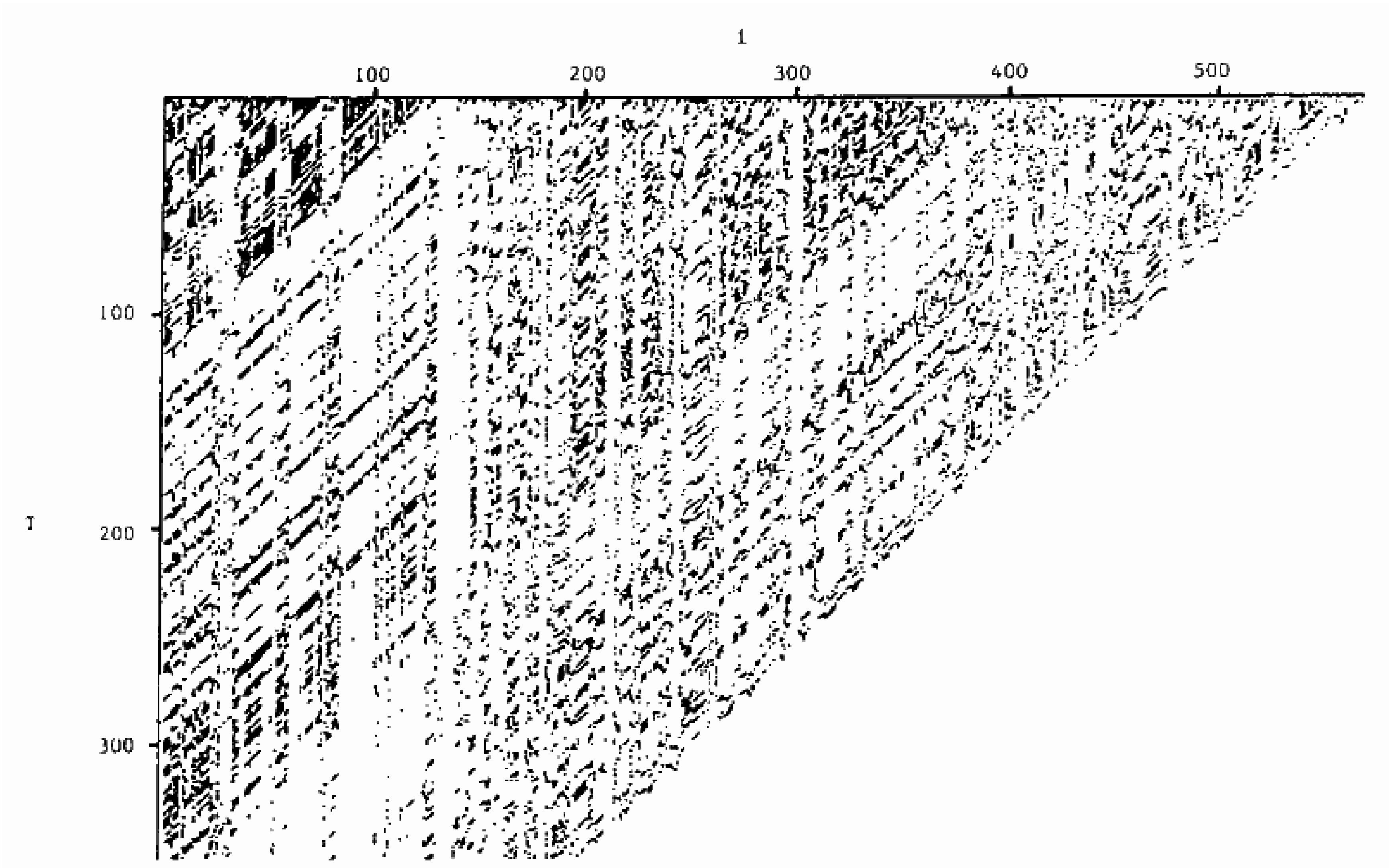
Logarithmic first differences are generally used in studying financial data, to handle nonstationarity.

This emphasizes noise component relative to signal.

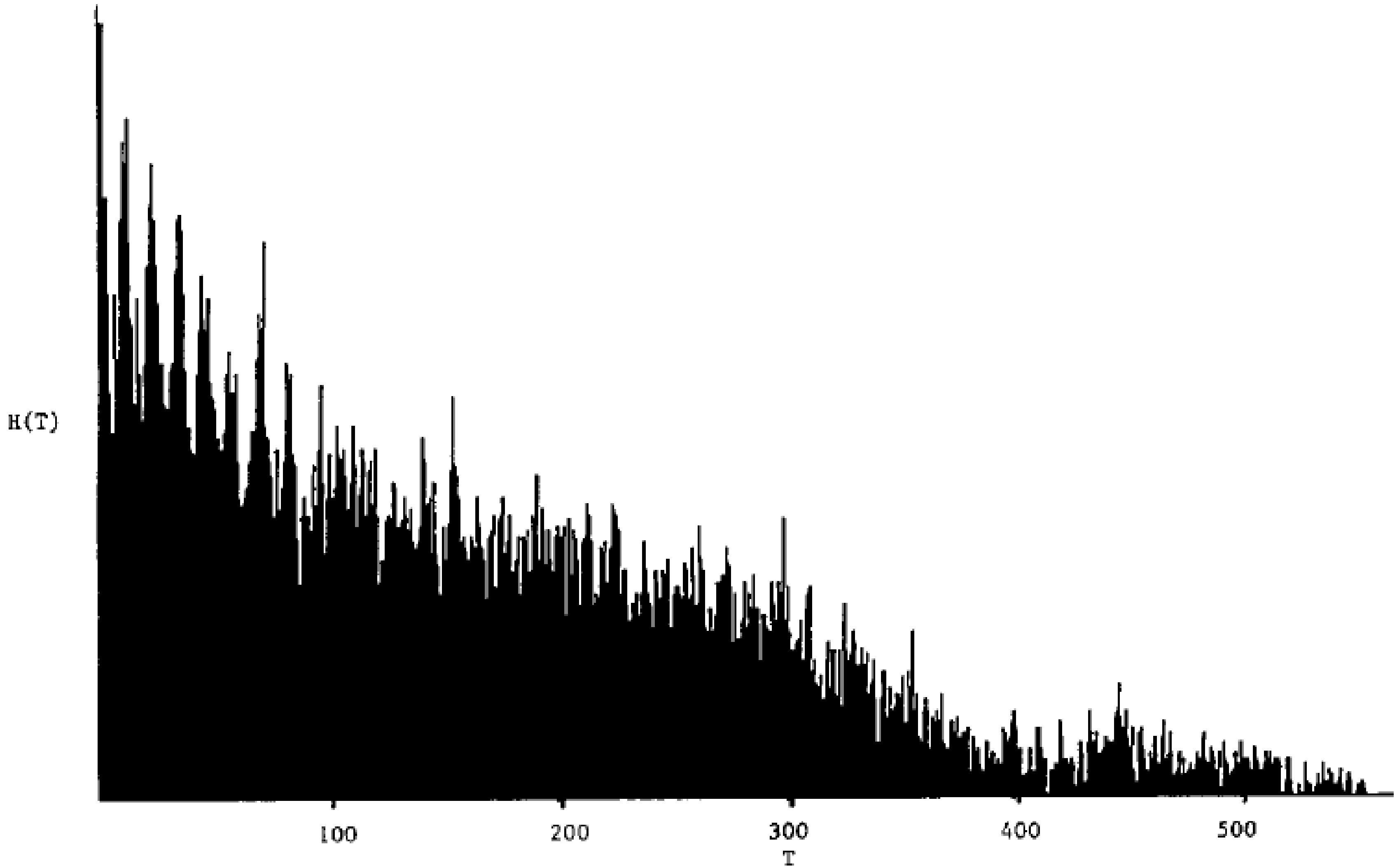
Therefore, several additional methods used to process data for purposes of testing robustness of close returns test.

Detrending methods

Series	Detrending method			
	Polynomial detrending	Log first differencing	Moving average	Low frequency filtering
Unemployment rate	x	x		x
Employment	x	x		x
Real GNP	x	x		x
GPDI	x	x		x
Industrial production	x	x	x	x
Work stoppages		x	x	x
Stock returns				x
Treasury bill returns		x	x	x
Exchange rates		x	x	x



Close returns plot of work stoppage data



Close returns histogram of work stoppage data.
Note prevalence of regularly spaced sharp peaks.