PHYS 431/750: Galactic Dynamics

Fall 2011, Homework #5

(Due Monday, December 5, 2011, 11 am)

1. (a) A singular isothermal sphere has $\rho \propto e^{-\phi/\sigma^2}$ and $\rho(r) \propto r^{-2}$. Show that the potential may be written as

   $$\phi(r) = v_c^2 \log_e r + \text{constant}\,.$$

   Compute the circular angular velocity Ω, the epicyclic frequency κ, and the pattern speed $\Omega_p = \Omega - \frac{1}{2}\kappa$ for this potential.

   (b) Prove that at any point in an axisymmetric system at which the local density is negligible, the epicyclic, vertical, and circular frequencies κ, ν, and Ω satisfy $\kappa^2 + \nu^2 = 2\Omega^2$.

2. Sparke & Gallagher, Problem 5.12.

3. Sparke & Gallagher, Problem 5.13.

4. *[Graduate students only]*

   (a) Given the dispersion relation for a gas disk (Binney & Tremaine Eq. 6-45)

   $$(\omega - m\Omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 v_s^2,$$

   verify the expression given by Binney & Tremaine (Equation 6-75) for the group velocity $v_g \equiv (\partial\omega/\partial k)_R$:

   $$v_g(R) = \text{sign}(k)\frac{|k|v_s^2 - \pi G\Sigma}{\omega - m\Omega}\,.$$

   (b) Show that, for a marginally stable disk with

   $$Q \equiv \frac{v_s \kappa}{\pi G\Sigma} = 1,$$

   the group velocity is (to within a sign) equal to the sound speed v_s.