PHYS 431/531: Galactic Dynamics

Fall 2011, Homework #2

(Due October 20, 2011)

1. (a) Given the definitions of the Oort constants A and B presented in class (Eqs. 2.13 and 2.16 in the text),

$$A = -\tfrac{1}{2} R \left(\frac{V}{R}\right)' \qquad B = -\tfrac{1}{2} \frac{(RV)'}{R},$$

verify that $A + B = -V'(R_0)$ and $A - B = V_0/R_0$, where $V(R)$ is the Galactic rotation law, R_0 is the distance from the Sun to the Galactic center, and $V_0 = V(R_0)$.

(b) Consider the spherically symmetric density distribution ρ given by

$$\rho(R) = \rho_0 \left(1 + \frac{R^2}{a^2}\right)^{-1}.$$

Derive an expression for the mass inside radius R. What is the circular orbital speed at radius R? Hence determine the form of $A(R)$ and $B(R)$ for $R \gg a$.

2. Air at sea level has density $\rho = 1.2$ kg/m^3 and sound speed $v_s = 330$ m/s.

(a) What is its Jeans length? What is the Jeans mass?

(b) By how much does the self-gravity of air change the frequency of a sound wave of wavelength 1 m?

(c) By what fraction (relative to the "short-wavelength" result $f = v_s/\lambda$) does the self-gravity of an interstellar cloud change the frequency of a sound wave with wavelength equal to half the Jeans length?

3. (a) A particle is dropped (from radius a with zero velocity) into the gravitational potential corresponding to a static homogeneous sphere of radius a and density ρ. Calculate how long the particle takes to reach the center of the sphere. [Hint: the equation of motion is $d^2r/dt^2 = -GM(r)/r^2$.]

(b) Calculate the time required for a homogeneous sphere of radius a and density ρ with no internal pressure support to collapse under its own gravity. [Apply the previous equation of motion to a particle on the surface.]

4. Interstellar gas in many galaxies is in virial equilibrium with the stars, in that the rms speed of the gas particles is the same as the rms stellar speed. Consider a large elliptical galaxy with a virial radius of 100 kpc and a mass of 10^{12} solar masses. Calculate the rms stellar velocity using the virial theorem. Hence estimate the temperature of the interstellar gas, assuming that it is composed entirely of hydrogen.