PHYS 501: Mathematical Physics I

Fall 2011

Homework #4
(Due: November 9, 2011)

1. Use contour integration to compute the integral.

$$I = \int_{-1}^{1} \frac{dx}{(a^2 + x^2)\sqrt{1 - x^2}} \,,$$

where a is real and the integrand has a branch cut running from -1 to 1. Sketch the contour you have chosen and carefully justify your reasoning to evaluate or neglect each portion of the total integral.

2. (a) Find the series solution of the equation

$$(1 - x^2)y'' - xy' + n^2 y = 0$$

that is regular at $x = 0$. Under what circumstances (for what values of n) does the series converge for *all* x?

(b) Find two linearly independent solutions of the equation

$$4x^2 y'' + (1 - p^2)y = 0\,.$$

3. Given that one solution of the differential equation

$$y'' - 2xy' = 0$$

is $y(x) = 1$, use the Wronskian development to find a second, linearly independent solution. Describe its behavior near $x = 0$.

4. Find the Green's function $G(x, x')$ for the equation

$$y'' - k^2 y = f(x)\,,$$

for $0 \le x \le L$, with $y(0) = y(L) = 0$. (Find G by solving the differential equation, not just as a formal sum over eigenfunctions!)

5. (a) A function $f(x)$ is periodic with period 2π, and can be written as a polynomial $P(x)$ for $-\pi < x < a$ and as a polynomial $Q(x)$ for $a < x < \pi$. Show that the Fourier coefficients c_n of f go to zero at least as fast as $1/n^2$ as $n \to \infty$ if $P(a) = Q(a)$ and $P(-\pi) = Q(\pi)$ (i.e. f is continuous), but only as $1/n$ otherwise.

(b) Of what function is

$$\sin x - \frac{\sin 3x}{9} + \frac{\sin 5x}{25} + \ldots$$

the Fourier series? Prove the result from the series—don't just plot the function and write down a formula!