

# PHYS 501: Mathematical Physics I

Fall 2011

Homework #3

(Due: October 19, 2011)

- (a) Sketch the image in the  $w$ -plane of the region  $\text{Re}(z) > A$ , under the mapping  $w = 1/z$ . Indicate on your sketch the images of (i) lines of constant  $\text{Re}(z)$  and (ii) lines of constant  $\text{Im}(z)$ . Be sure to distinguish between the cases  $A > 0$  and  $A < 0$ .  
(b) Repeat part (a) for the mapping  $w = e^z$ .  
(c) Locate the branch points, draw all distinct sets of branch cuts, and discuss the continuity of the function

$$w(z) = \sqrt{z(z^2 + 1)}.$$

- (a) Find *all* Laurent or Taylor expansions of the function

$$f(z) = \frac{z}{z^2 - 1}$$

about the point  $z = 2$ , i.e. expand the function as a series of the form

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z - 2)^n.$$

Note that there are several different regions in which different expansions apply.

(b) Fill in the blanks, where  $u(x, y)$  and  $v(x, y)$  are the real and imaginary parts, respectively, of the analytic function  $w(z)$ :

- $u(x, y) = e^x \cos y$ ,  $v(x, y) = ?$ ,  $w(z) = ?$
- $u(x, y) = ?$ ,  $v(x, y) = y(3x^2 - y^2 - 1)$ ,  $w(z) = ?$
- $u(x, y) = ?$ ,  $v(x, y) = ?$ ,  $w(z) = \tan^{-1} z$ .

- Evaluate the following integrals, all taken counterclockwise around the circle  $|z| = 1.5$

$$(a) \oint \frac{\cos z}{z} dz \quad (b) \oint \frac{\tan z}{z} dz \quad (c) \oint \frac{2z^2 + 3z - 1}{z - 1 + i} dz \quad (d) \oint \frac{dz}{(z - 2)^2}$$

- Evaluate the following integrals using the residue theorem:

$$(a) \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^3} \quad (b) \int_0^{\infty} \frac{x \sin x}{x^2 + 1} dx$$
$$(c) \int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx \quad (d) \int_{-\infty}^{\infty} \frac{x^2 e^x}{1 + e^{2x}} dx,$$

where  $a$  and  $b$  are nonzero and  $a \neq b$ . In each case, sketch the contour you choose and clearly quote all theorems used in the derivation of your results.