PHYS 431/531: Galactic Dynamics

Fall 2011, Homework #1

(Due October 6, 2011)

1. Assume that the Galaxy is 10 Gyr old, the rate of star formation in the past was proportional to $e^{-t/T}$, where t is time since the Galaxy formed and $T = 3$ Gyr, and stellar lifetimes are given by

$$t(M) \;=\; 10\,\mathrm{Gyr}\,\left(\frac{M}{M_\odot}\right)^{-3}\,.$$

   Calculate the fractions of all (a) $2M_\odot$ and (b) $5M_\odot$ stars ever formed that are still around today.

2. (a) A close (i.e. unresolved) binary consists of two stars each of apparent magnitude m. What is the apparent magnitude of the binary?

   (b) A star has apparent magnitude $m_V = 10$ and is determined spectroscopically to be an A0 main sequence star. What is its distance? (See Sparke & Gallagher Table 1.4.)

3. (a) If the mass function for stars follows the Salpeter distribution, with

$$\xi(M) \equiv \frac{dN}{dM} = A\,M^{-2.35}$$

   (where dN is the number of stars with masses between M and $M + dM$; see Sparke & Gallagher, p. 66), for $M_l < M < M_u$, with $M_l \ll M_u$, and the stellar mass–luminosity relation is

$$L(M) \;\propto\; M^4\,,$$

   show that the total number and total mass of stars depend mainly on M_l, while the total luminosity depends mainly on M_u. Specifically, for $M_l = 0.2M_\odot$ and $M_u = 100M_\odot$, calculate the masses M_1 and M_2 such that 50% of the total mass is contained in stars with $M < M_1$, while 50% of the total luminosity is contained in stars with $M > M_2$.

   (b) Astronomers often approximate the stellar mass function $\xi(M)$ by a Salpeter power-law with a low-mass cutoff, but the Kroupa distribution

$$\xi(M) = \begin{cases} C\,M^{-0.3} & (M < 0.1M_\odot) \\ B\,M^{-1.3} & (0.1M_\odot < M < 0.5M_\odot) \\ A\,M^{-2.35} & (M > 0.5M_\odot) \end{cases}$$

   is actually a better description (A is the same as in part (a) and the other constants of proportionality are chosen to ensure that ξ is continuous.) If the upper mass limit in all cases is $M_u = 100M_\odot$ and we assume the same simplified mass–luminosity relation as in part (a), what low-mass cutoff M_l must be chosen in order that the truncated power-law has the same (i) total number of stars, (ii) total mass, and (iii) total luminosity as the Kroupa distribution?

4. (a) Use Gauss's law to derive an expression for the gravitational force in the z direction due to an infinite sheet of surface density Σ lying in the x–y plane.

   (b) A star has velocity 30 km/s perpendicular to the Galactic plane as it crosses the plane, and is observed to have a maximum departure above the plane of 500 pc. Approximating the disk as an infinite gravitating sheet of matter, estimate its surface density Σ (i) in $\mathrm{kg\,m^{-2}}$ and (ii) in $M_\odot\,\mathrm{pc}^{-2}$.