Modeling Three- and Four- Coupled Phase Qubits

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Introduction

The Josephson junction phase qubit has been shown to be a viable candidate for quantum computation [1-3]. In recent years, the two-coupled phase qubit system has been extensively studied theoretically and experimentally [4-5]. The theory of entanglement in tripartite systems has also been studied in [6]. Coupled flux qubits have been studied in [7-9] and multiple flux qubits in [10-11].

Building a working quantum computer requires the ability to deliberately entangle multiple qubits in a controlled fashion. Our research focuses on manipulating multiply-coupled phase qubits into specific entanglements of single-qubit states by control of bias currents.

We have analyzed the quantum behavior of systems of three and four capacitively-coupled phase qubits with different possible configurations. Using a harmonic oscillator model, we numerically estimate:

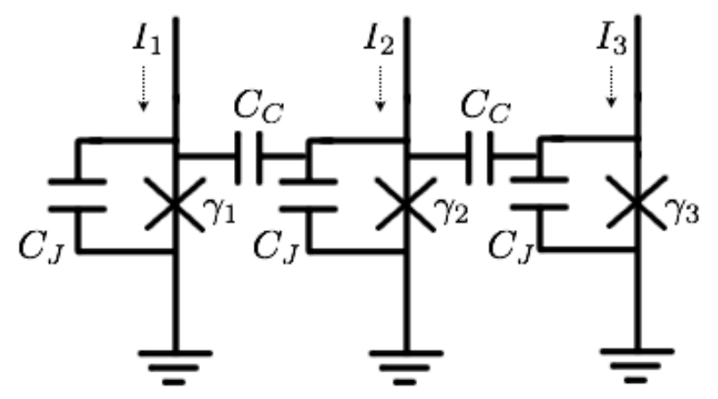
1. First-excited energy levels and eigenstates for each configuration as a function of variable biasing currents.

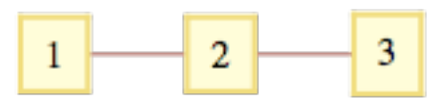
2. Time evolution of an arbitrary initial system state at fixed biasing currents for these systems.

Theory

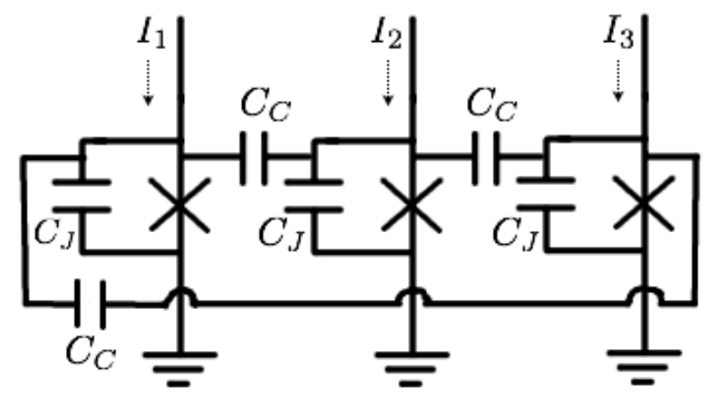
Any circuit consisting of Josephson junctions connected by identical capacitors can be represented by a (simple, undirected) graph in which each vertex corresponds to a junction and each edge to a capacitor:

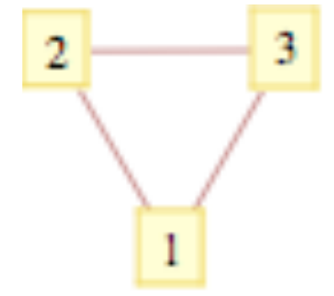
3-junction linear configuration



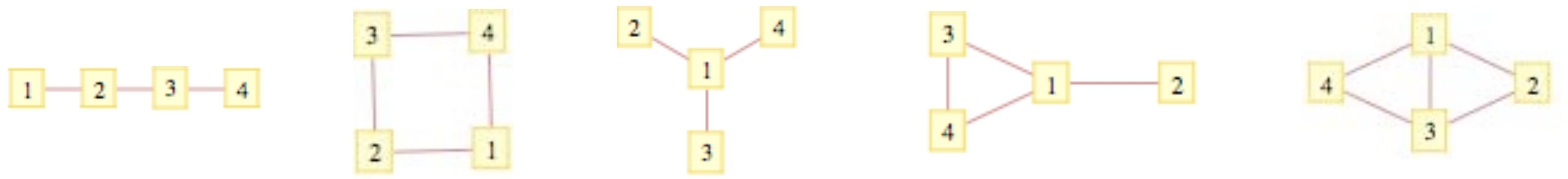


3-junction cyclic configuration





4-qubit configurations we have studied include the following circuits:



As shown in [10], the Hamiltonian for such a system can be written

$$\hat{H} = \frac{1}{2}\left(C_J\left(\frac{\Phi_0}{2\pi}\right)\right)^{-2}[\hat{p}_1, \hat{p}_2, \cdots]\left[\quad C \quad\right]\begin{bmatrix}\hat{p}_1\\ \hat{p}_2\\ \vdots\end{bmatrix} + \hat{W}_1(\gamma_1) + \hat{W}_2(\gamma_2) + \cdots$$

where junction momentum, junction voltage, and washboard potential are

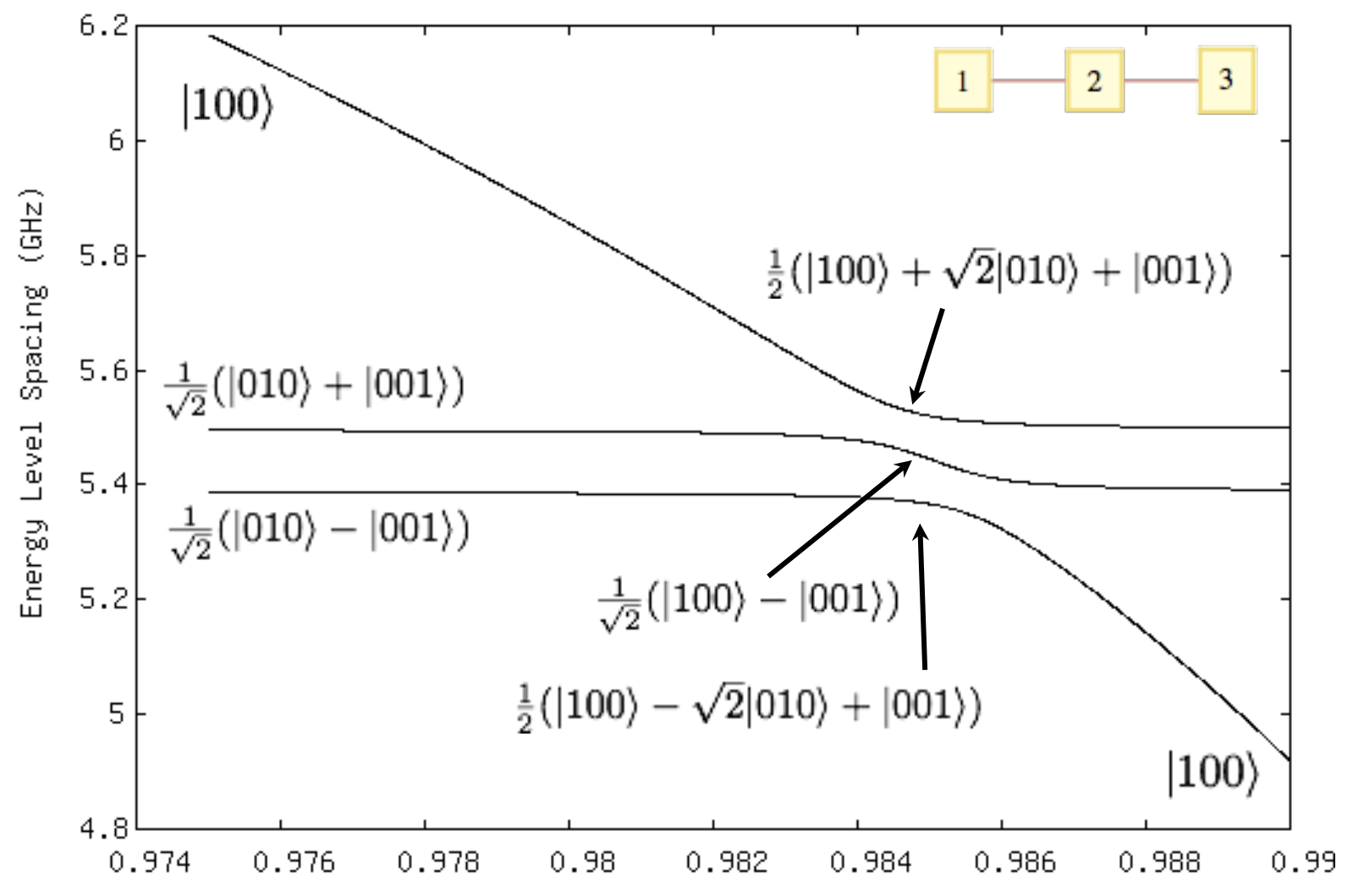
$$\hat{p}_n \equiv C_J\left(\frac{\Phi_0}{2\pi}\right)\hat{V}_n \qquad \hat{V}_n = \left(\frac{\Phi_0}{2\pi}\right)\dot{\gamma}_n \qquad \hat{W}_n \equiv -\left(\frac{\Phi_0}{2\pi}\right)\left(I_0\cos(\gamma_n) + I_n\gamma_n\right)$$

here $C_{jk} = C_J\delta_{jk} + C_C\mathcal{L}_{jk}$, where $[\ \mathcal{L}\ ]$ is the circuit's Laplacian matrix, and the conjugate momentum operators $\hat{p}'_j$ are found $\hat{p}'_j = \frac{1}{C_J}\sum C_{jk}\hat{p}_k$.

We find eigenstates of this Hamiltonian on the basis $\{\ |001\rangle, |010\rangle, |001\rangle\ \}$. Time evolution of an initial state is found by projecting it onto these eigenstates, each of which evolves only by a phase factor $\exp(-\frac{i}{\hbar}E_n t)$.

Discussion of Results

Fig. 1. 3-qubit linear configuration



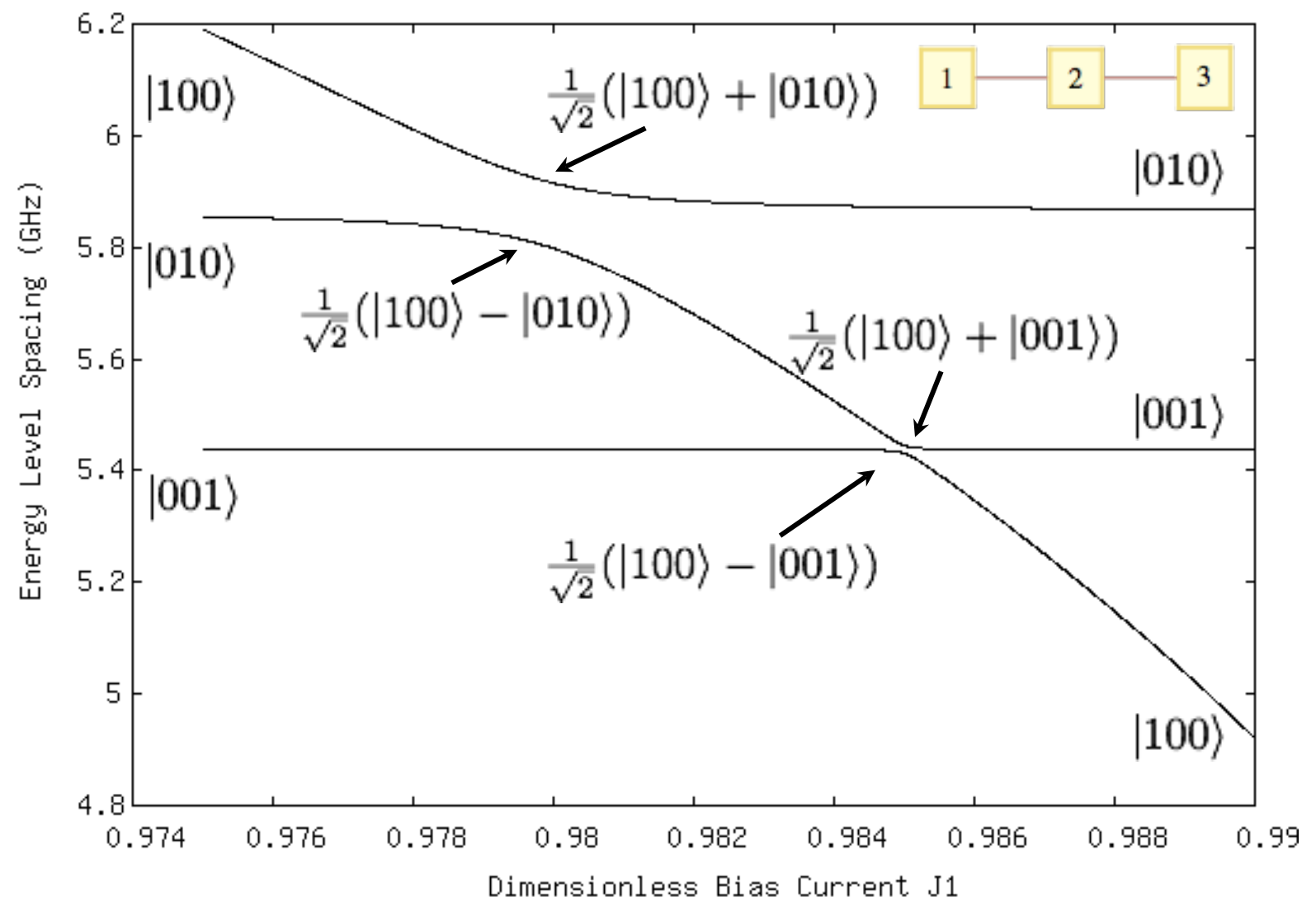
- Fig. 1 shows estimated energy levels for the first-excited states of the 3-qubit linear system with bias currents *J2* = 0.9844, *J3* = 0.985 .

- Here *J1* is increased while *J2* and *J3* are held fixed. Avoided level crossings are found near *J1* = 0.985 as the individual junction wavefunctions become entangled.

- Fig. 2 shows estimated energy levels for the same system, but now the bias current through qubit 2 detuned to *J2* = 0.979 .

- Qubits 1 and 3 couple more strongly to qubit 2 than to each other. As a result, the avoided crossing between qubits 1 and 3 is much less obvious that between qubits 1 and 2.

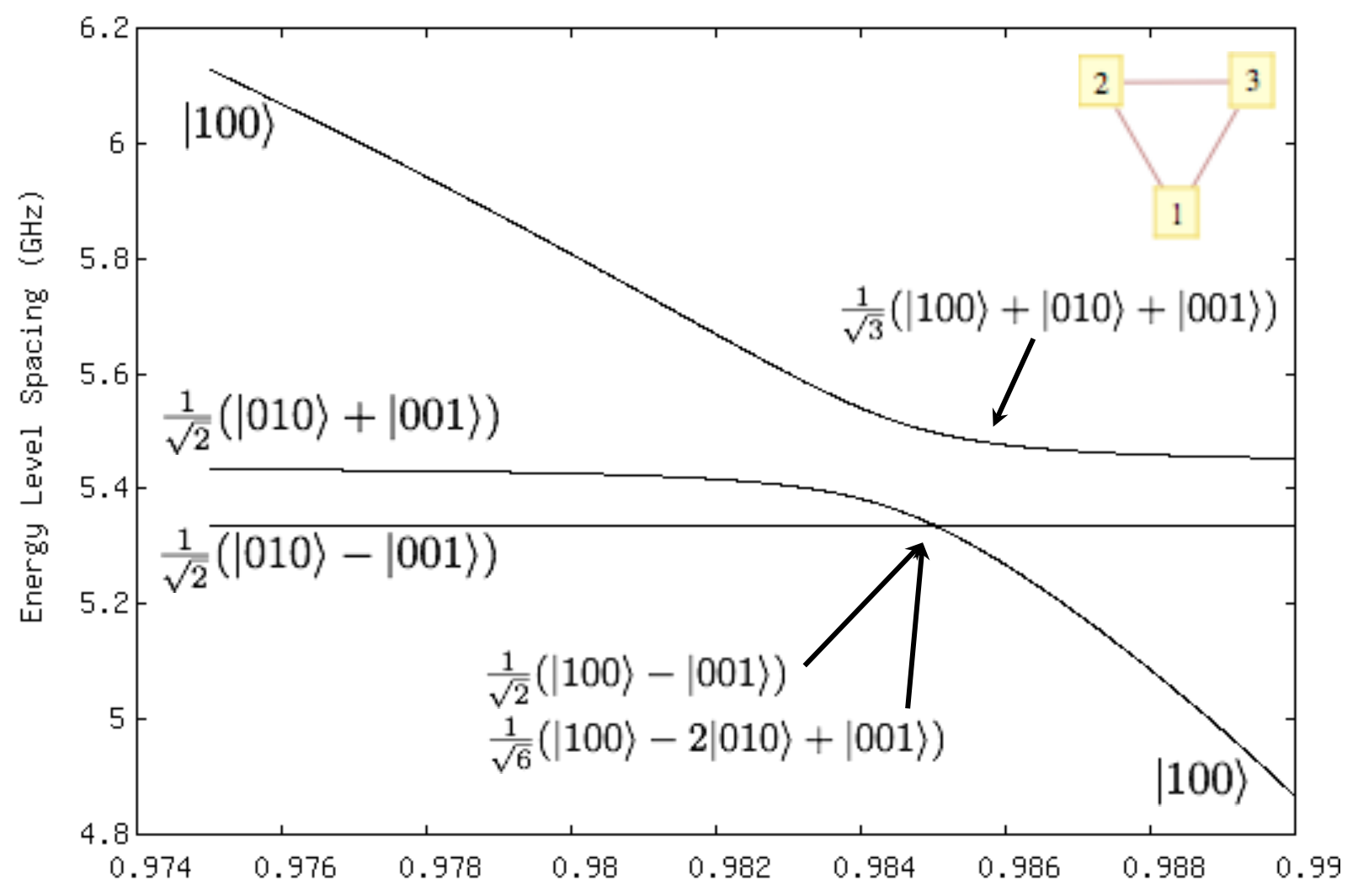
Fig. 2. 3-qubit linear configuration (unequal bias currents)



- Fig. 3 shows first-excited energy levels for the 3-qubit cyclic configuration as *J1* varies. (*J2* and *J3* are again held fixed at 0.985 .)

- When *J1=J2=J3*, the system becomes 2-fold degenerate. This "non-avoided crossing" is a result of symmetry properties of the system.

Fig. 3. 3-qubit cyclic configuration



- Fig. 4 shows the same system, except *J2* is detuned and held fixed at 0.9844 .

- The slight detuning of one qubit breaks the symmetry of the system, lifting the degeneracy and changing the entangled eigenstates.

- Junction entanglement can be significantly altered by small variations in bias current.

Fig. 4. 3-qubit cyclic configuration

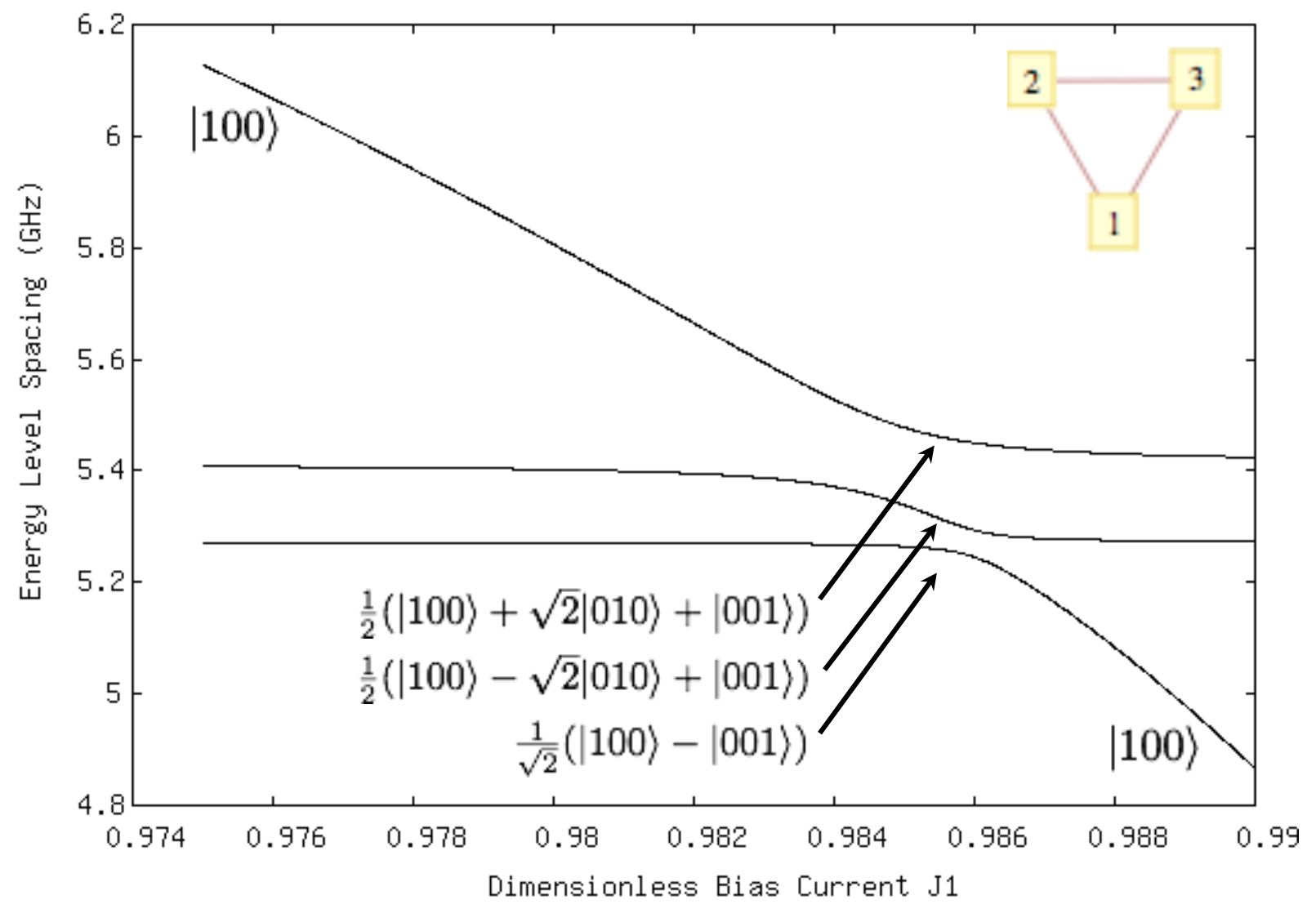
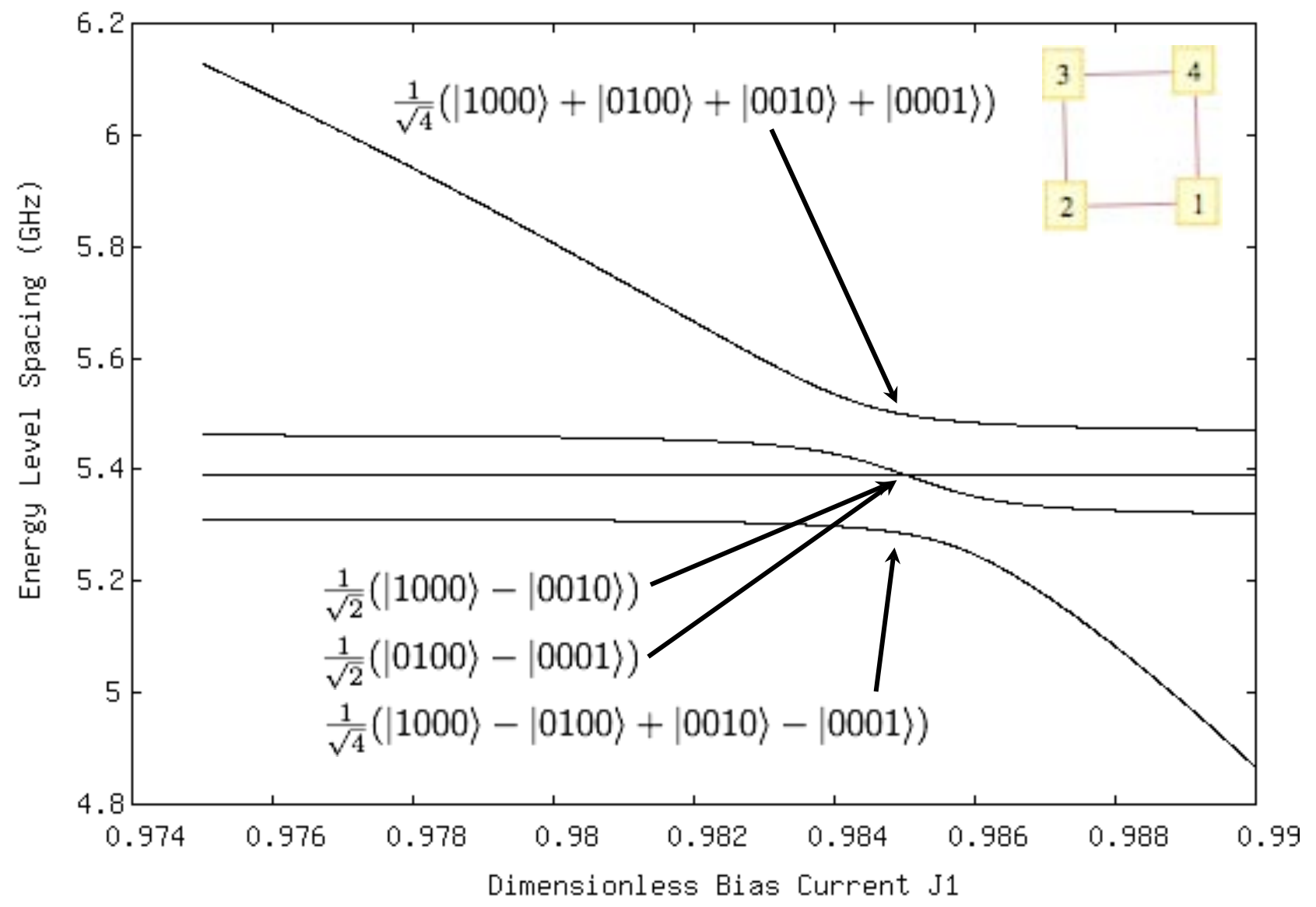


Fig. 5. 4-qubit cyclic configuration



- Fig. 5 shows energy levels for the 4-qubit cyclic system with bias currents *J2=J3=J4=* 0.985 .

- At equal bias currents, this system is 2-fold degenerate.

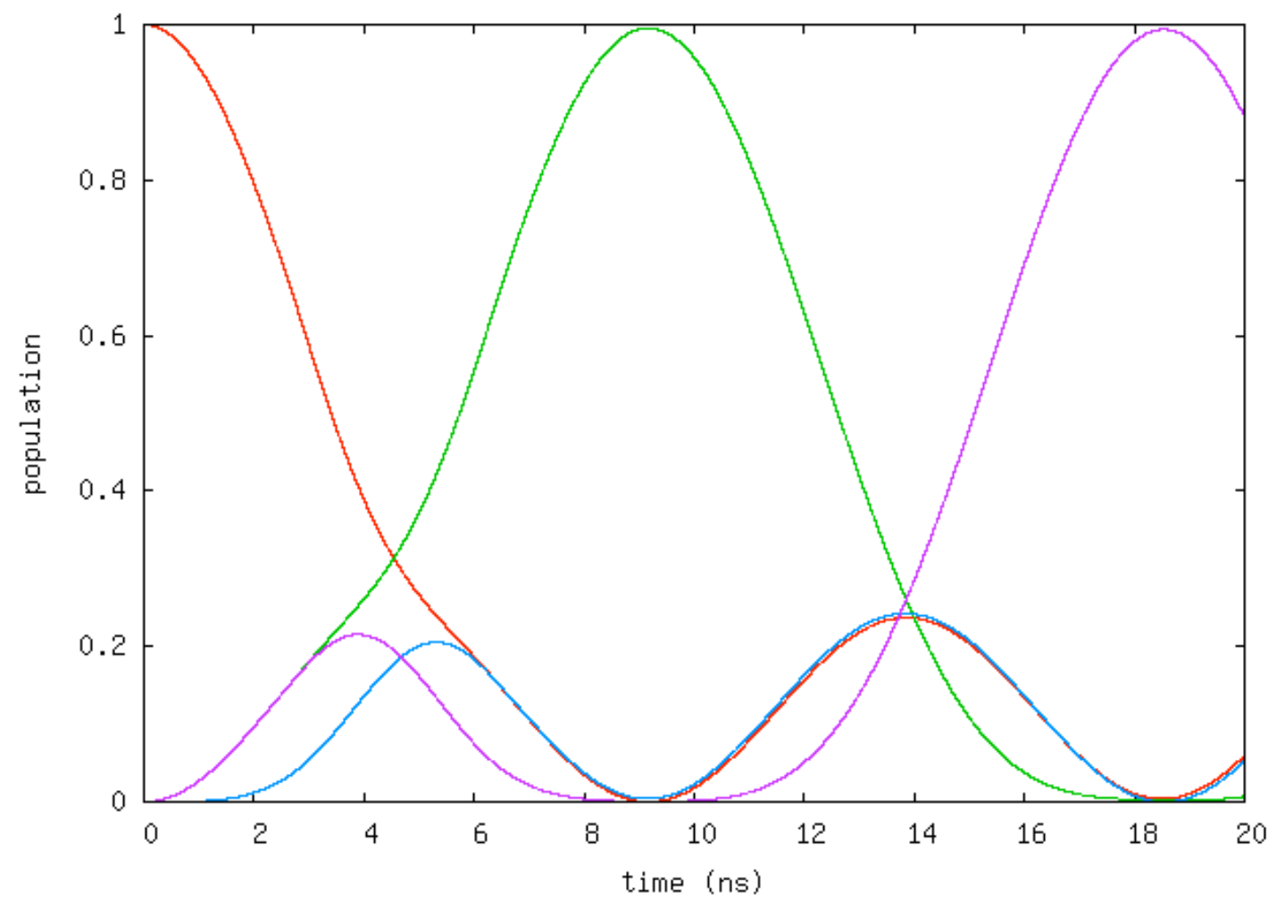
- As before, small changes in bias current result in broken symmetry and no degeneracy.

- Fig. 6 shows time-evolution of the population of individual qubits. The initial state is $\Psi(0) = |1000\rangle$. The state population is directed to qubit 2 by momentarily detuning qubits 1 and 2.

- By detuning one or more junctions, state transfer from any one qubit to any other is possible.

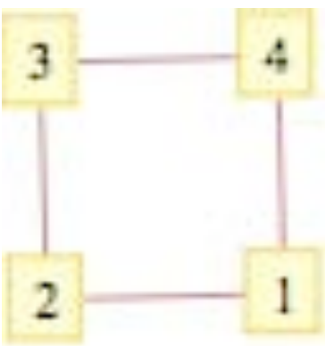
- Detailed knowledge of such evolutions is needed e.g. for transfer of a qubit state in a quantum logic or memory circuit.

Fig. 6. State transfer in 4-qubit cyclic configuration

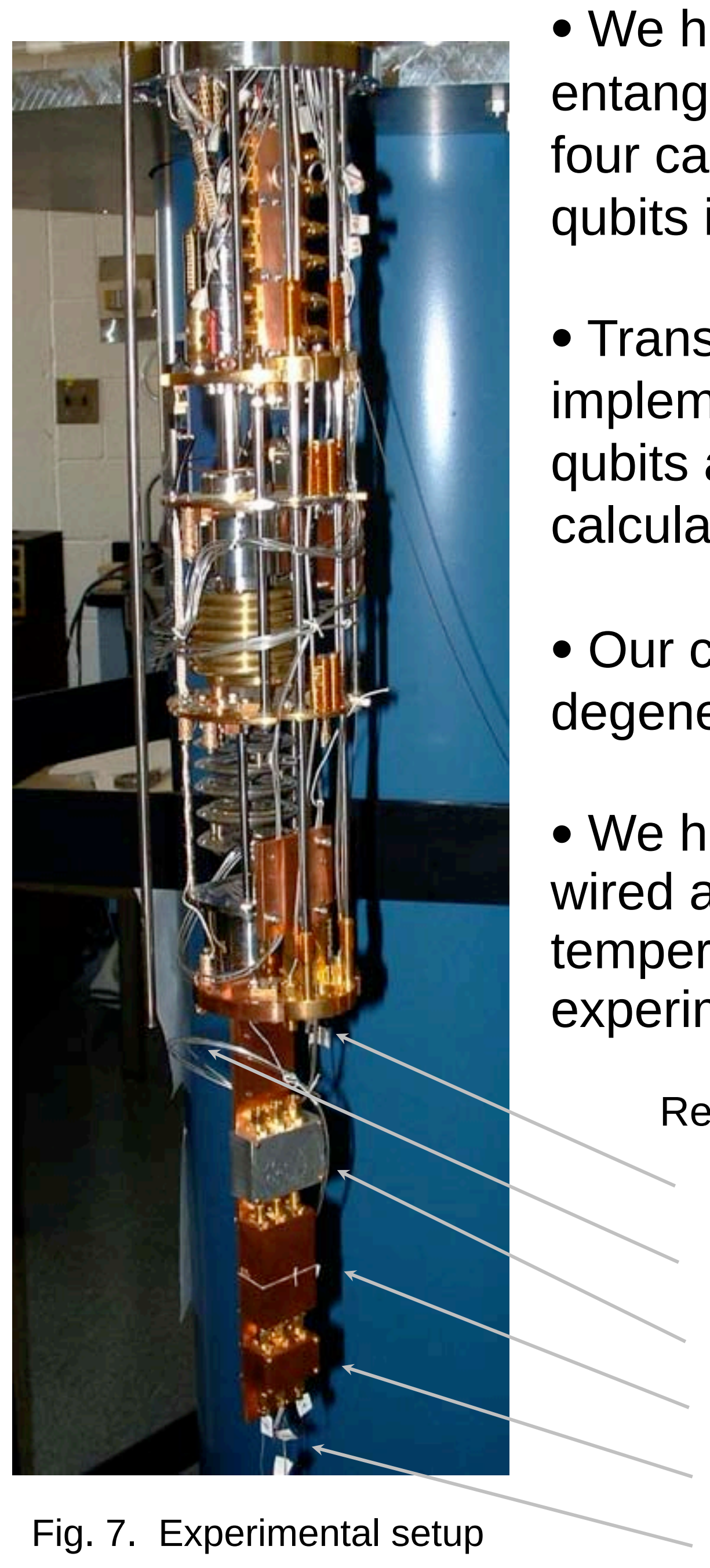


Population (y-axis) refers here to the value of $||\langle a|\Psi(t)\rangle||^2$ where:

Red $\Rightarrow |a\rangle = |1000\rangle$
Green $\Rightarrow |a\rangle = |0100\rangle$
Blue $\Rightarrow |a\rangle = |0010\rangle$
Purple $\Rightarrow |a\rangle = |0001\rangle$



Summary and Future Work



- We have calculated the energy levels of entangled states corresponding to three and four capacitively-coupled Josephson phase qubits in different geometries.

- Transfer of quantum states can be implemented dynamically by tuning/detuning qubits at avoided crossings determined by the calculated energy levels.

- Our calculations have identified energy degeneracies intrinsic in symmetric systems.

- We have developed instrumentation and wired a dilution refrigerator with a base temperature of 10 mK for multi-qubit experiments to verify our simulations (Fig. 7).

Recent improvements are shown in Fig. 7 :

microwave line
thermocoax cable filters
sample mount
copper-powder filters
LC filters
bias-current lines

Fig. 7. Experimental setup

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