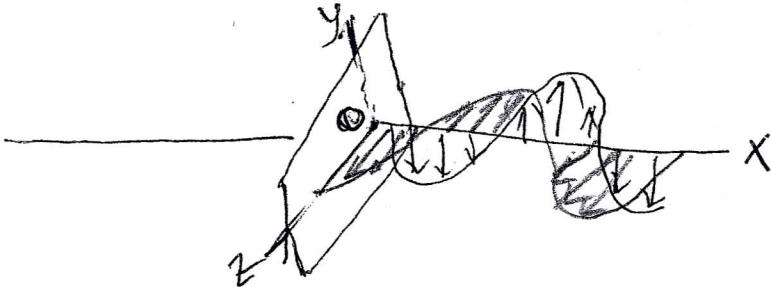


Standing Electromagnetic Waves

Electromagnetic waves can get reflected from the surface of a conductor or of a dielectric (glass). The incident and the reflected waves can superpose to form standing waves very much like the mechanical waves on a string.



Suppose a sinusoidal wave traveling to the left gets reflected from a conductor at $x=0$. The incident and reflected E-wave and B-wave form standing waves with nodes and antinodes as shown in the above figure (Fig. 32.22) and can be represented as

$$E_y(x,t) = E_{\max} [\cos(kx + \omega t) - \cos(kx - \omega t)]$$

$$B_z(x,t) = B_{\max} [-\cos(kx + \omega t) - \cos(kx - \omega t)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

we get

$$E_y(x,t) = -2E_{\max} \sin kx \sin \omega t$$

$$B_z(x,t) = -2B_{\max} \cos kx \cos \omega t$$

These equations are similar to the standing wave equation we derived for the stretched string. We see that E-field will have nodes at $kx = 0, \pi, 2\pi, \dots \Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$ since $k = \frac{2\pi}{\lambda}$ and antinodes at $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$

Standing Waves in a Cavity.

In a cavity of length L like a microwave oven $E_y = 0$ at $x = 0 + L$

$$\Rightarrow \sin kL = 0 \Rightarrow kL = \pi, 2\pi, 3\pi, \dots = n\pi$$

$$\Rightarrow \frac{2\pi}{\lambda_n} L = n\pi \Rightarrow \lambda_n = \frac{2L}{n}, (n=1, 2, \dots)$$

$$f_n = \frac{c}{\lambda_n} = n \frac{c}{2L} \quad (n=1, 2, \dots)$$

The Nature and Propagation of LightNature of Light

1. Corpuscular or Particle Nature - Proposed by Newton

Could explain ① straight line propagation, ② law of reflection

Could not explain ① law of refraction ② Interference, diffraction

2. Wave Nature (Work of Huygens, Young, Fresnel, Maxwell)

Explains reflection, refraction, interference, diffraction and polarization

Maxwell's Theory showed that light is an electromagnetic wave. Work of Heinrich Hertz conclusively proved that light is indeed an electromagnetic wave.

During the early part of the 20th century ~~most~~ some experiments involving absorption or emission of light showed that light can have a particle nature and light waves can carry bundles of energy called photons, the quantum of light.

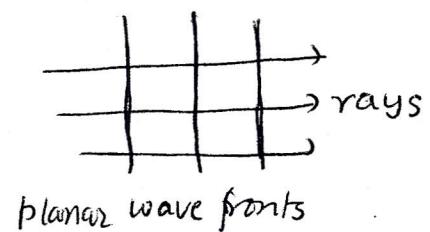
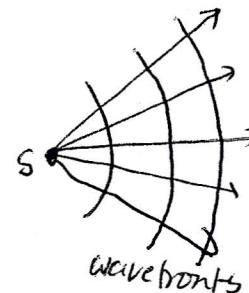
In this chapter we will discuss the wave nature of light and its consequences.

Wave Fronts and Rays

A wave front is a surface of constant phase. If you have a point source, light will propagate in all directions as shown in the figure. If you draw a surface of constant phase it will be spherical as shown. The straight arrows show the direction of propagation and are called rays.

Far away from the source, the radii of the wavefronts will be large and the wave front can be considered as a plane surface as shown. Here the rays are parallel.

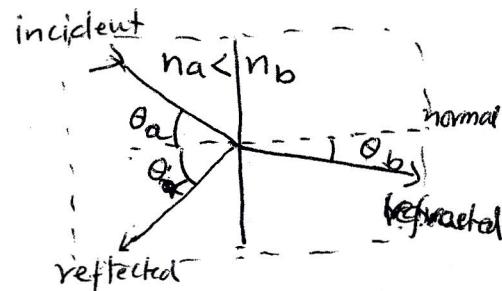
We first consider geometrical optics where light can be represented by rays.



We have seen that when light enters a material medium its speed is lowered but its frequency remains unchanged. Since $\lambda = \frac{c}{f} < \lambda_0$, the wavelength of light in a medium will be lowered. Here λ_0 is the wavelength in vacuum. We have defined the refractive index of the medium $n = \frac{c}{v} > 1$ for any medium. $n_{\text{glass}} \approx 1.5$, $n_{\text{water}} = 1.33$. Note $n = \frac{c}{v} = \frac{\lambda_0 f}{\lambda_f} = \lambda_0 / \lambda$.

Laws of Reflection and Refraction

When a ray of light strikes a smooth interface of two transparent media (like air and glass or water and glass), part of it is reflected and a part is refracted or transmitted as shown in the figure.



Experiments show That

- ① The incident, reflected and refracted rays and the normal to the surface all lie on the same plane
- ② The angle of incidence is equal to the angle of reflection $\theta_r = \theta_a$ Law of reflection
- ③ The sines of the angle of incidence and the angle of refraction are related to each other according to

$$n_a \sin \theta_a = n_b \sin \theta_b$$

or

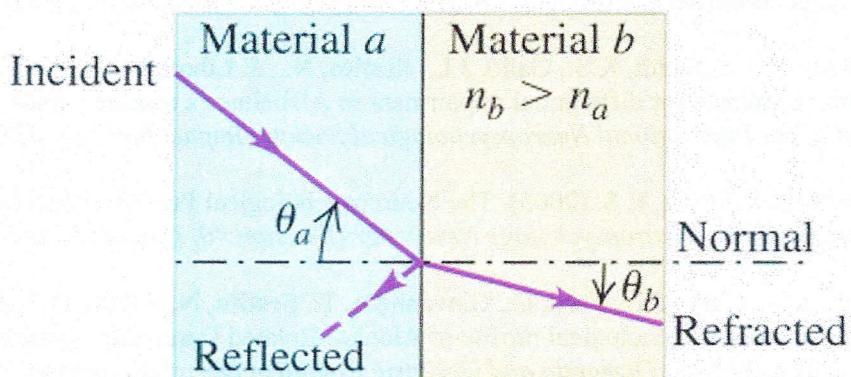
$$\frac{\sin \theta_a}{\sin \theta_b} = \frac{n_b}{n_a}$$

Law of refraction or
Snell's law

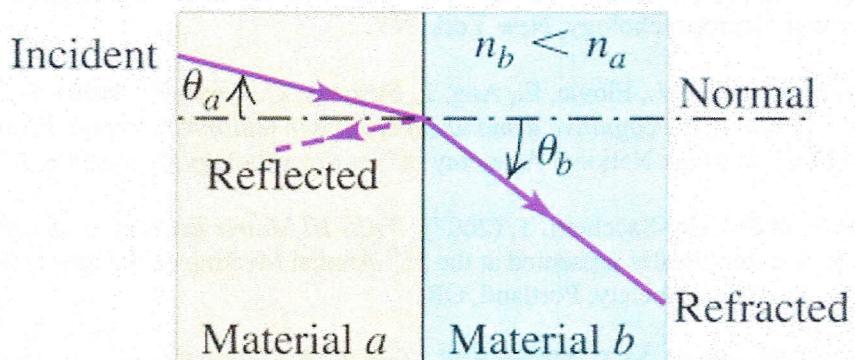
These experimental observations can be derived from wave theory.

175. Figure 33.8 Refraction and reflection in three cases

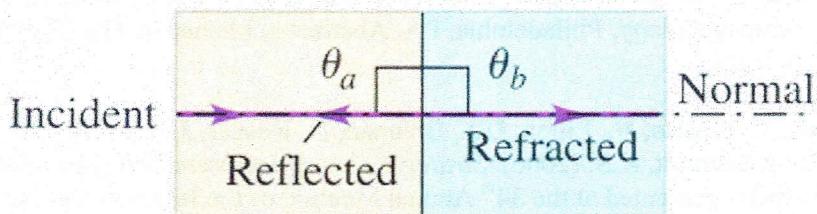
(a) A ray entering a material of *larger* index of refraction bends *toward* the normal.



(b) A ray entering a material of *smaller* index of refraction bends *away from* the normal.



(c) A ray oriented along the normal does not bend, regardless of the materials.



Total Internal Reflection

5

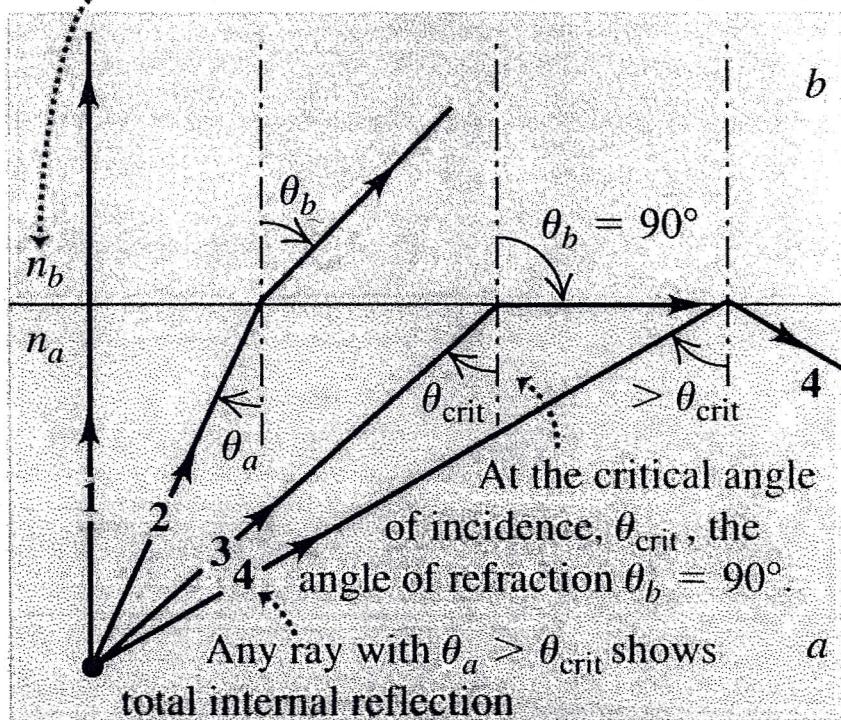
Under certain circumstances all of the light can be reflected back from the interface, with none of them being refracted (transmitted) even though the second material is transparent. Examine the following figure which shows several incident rays radiating from a single point source. In this case we assume $n_b < n_a$ so that $\theta_b > \theta_a$. As the angle of incidence increases θ_b also increases. At a certain critical value of $\theta_a = \theta_{\text{crit}}$, $\theta_b = 90^\circ$. In this case $\sin \theta_{\text{crit}} = \frac{n_b}{n_a}$ from Snell's law.

For glass to air transition $\theta_{\text{critical}} = \sin^{-1} \frac{n_{\text{air}}}{n_{\text{glass}}} = \sin^{-1} \frac{1}{1.52} = 41.1^\circ$

Total internal reflection will occur when $\theta_a > \theta_{\text{critical}}$

(a) Total internal reflection

Total internal reflection occurs only if $n_b < n_a$.

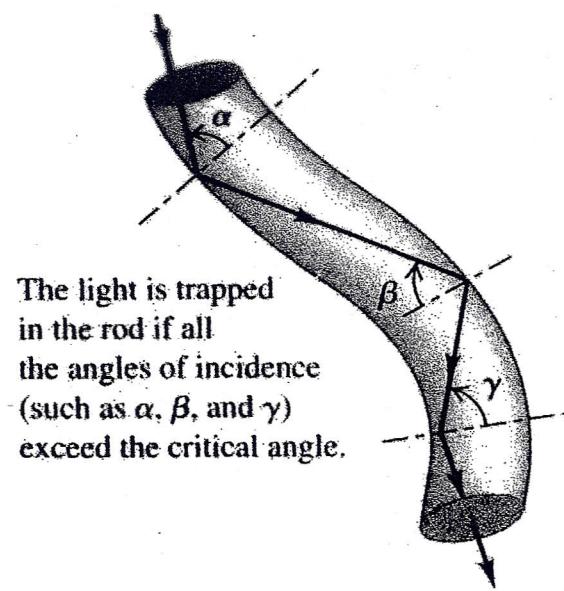


Total Internal reflection has many applications:

- ① Porro Prism ② light pipe ③ Fiber optics (4) endoscopes, etc. For details read text.

In the case Porro prism, the incident light is incident at 45° on the 45° faces and get totally reflected. No mirror will reflect light totally. Such a reflector is permanent and not affected by tarnishing. Binoculars use combination of two Porro prism. See figure (next page)

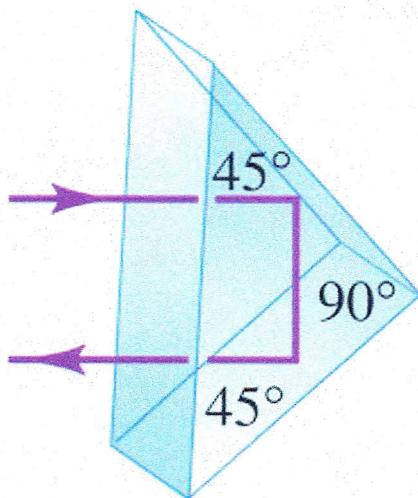
When a light beam enters at one end of a transparent rod, it can get totally reflected internally and the light is trapped within the rod. A bundle of such flexible rods can transmit images, information as modulated laser beams. They have medical applications as endoscopes (to study bronchial tubes, bladder, the colon and so on). Can be enclosed in hypodermic needles for study of tissues and blood vessels.



176. Figure 33.14a Total internal reflection

(a) Total internal reflection in a
Porro prism

since $\theta_c = 41.1^\circ$, total
internal reflections occur
on 45° faces



If the incident beam is oriented as shown, total internal reflection occurs on the 45° faces (because, for a glass-air interface, $\theta_{\text{crit}} = 41.1^\circ$).